HAND WRITTEN NOTES:-
OF
ELECTRICAL ENGINEERING

- SUBJECT:-
ELECTROMAGNETIC

THEORY
Electromagnetism:

- Electrostatics
  - Static electrical fields
    \( \mathbf{E}, \mathbf{B} \neq f(t) \)
  - Time varying electrical & magnetic fields
    \( \mathbf{E}, \mathbf{B}, \mathbf{B}, \mathbf{H} \neq f(t) \)
- Static electrical field intensity
  \( \text{N/C} ; \text{v/m} \)

Electrical field intensity:
- Electric displacement vector (C/m²)
  \( \mathbf{D} = \varepsilon \mathbf{E} \)
  - Permittivity
    \( \varepsilon_r \geq 1 \)
    - \( \varepsilon_r = 1 \) for free space,
      \( > 1 \) for any other dielectric

  \[ \varepsilon_r = \frac{\varepsilon}{\varepsilon_0} \]

- Dielectric constant
  \( \varepsilon_0 = 8.854 \times 10^{-12} \text{ F/m} \)

  \[ \varepsilon_0 = \frac{1}{36\pi} \times 10^{-9} \text{ F/m} \]

  \( \varepsilon_r = \frac{\varepsilon}{\varepsilon_0} \)

- Relative permittivity
  - Dielectric constant
    \( \text{unitless} \)

- Static magnetic fields
  \( \mathbf{B}, \mathbf{H} \neq f(t) \)

- Magnetic field intensity
  \( \text{Wb/m}^2 \)

- Magnetic field intensity
  \( \text{Tesla} (\text{T}) \)

- Magnetic flux density
  \( \text{B} \)

- Magnetic flux density
  \( \mathbf{B} \neq f(t) \)

Constant, irrespective of type of medium.
Statics Magnetic Fields:

\[ \mathbf{B} = \mu_0 \mathbf{H} \]

Magnetic field intensity \( \mathbf{H} \) (\( \text{A/m} \))
Magnetic flux density \( \mu_0 \mathbf{B} \) (\( \text{Wb/m}^2 \text{T} \))

\[ \mu = \mu_0 \mu_r \]

\( \mu_0 = \) permeability of free space
\[ \mu_0 = 4\pi \times 10^{-7} \quad \text{H/m} \]

\( \mu_r = \) relative permeability of medium

\[ \mu_r \leq 1 \]
unit less

\( \mu_r < 1 \quad \text{diamag.} \)
\( \mu_r = 1 \quad \text{non-mag.} \)
\( \mu_r > 1 \quad \text{paramag.} \)
\( \mu_r >> 1 \quad \text{bittermag.} \)

Unless specified \( \mu_r \approx 1 \)

\[ \begin{align*}
\text{Wax} & (\text{diamag.}); \quad \mu_r = 0.9933331 \\
\text{Air} & (\text{paramag.}); \quad \mu_r = 1.0000006 \\
\text{Cobalt} & (\text{bittermag.}); \quad \mu_r \approx 250 \\
\text{Fe} (0.5\% \text{ impure}); \quad \mu_r \approx 5000 \\
\text{Fe} (0.05\% \text{ impure}); \quad \mu_r \approx 200000 \\
\end{align*} \]
Time Varying elect. & mag. fields :=
\[ \mathbf{E}, \mathbf{D}; \mathbf{B}, \mathbf{H} = f(t) \]

Important points :

1. Maxwell's equations are a set of four eqns.
   Which a Relationship btw time varying elec. & mag. fields.
2. When ever any wave propagate then the elec.
   field, mag. field, & direction of propagation are
   mutually perpendiculars each other.

\[ (\mathbf{E} \perp \mathbf{H}) \perp \text{direction of propagation} \]

\[ \downarrow \]

TEM wave
Transverse EM waves (uniform plane waves)

3. When there is large mismatching btw. the length
   of filament & wavelength of operation at
   low freq. then entire power dissipation in element.
4. At high freq. the length is comparable to the
   wavelength of operation, then the power is
   radiated through that element.

5. At low freqs the depth of penetration is high.
   Therefore we use thick conductor, whereas at
   high frequencies depth of penetration is
   low & therefore thin conductors are used

The power from the transmitters to the earth is
transported with the help of co-axial
transmission line or a power sub-
transmission line.
Transmission line:

1. Parallel wide transmission line

\[ \frac{dI}{dz} = V \]

Co-axial T-L

\[ \text{used in CATV as CRO load} \]

Distributed parameter equivalent ckt. of T-L

\[ R \cdot \frac{dI}{dz} + L \cdot \frac{dV}{dz} = 0 \]

\[ C_1 \cdot \frac{dV}{dz} = \frac{1}{C_{\text{at}}} \]

For lossy line

\[ \begin{cases} R &= \frac{\Omega}{m} \\ L &= \frac{H}{m} \\ C &= \frac{F}{m} \\ G_1 &= \frac{S}{m} \end{cases} \]

Loss less line

\[ \begin{cases} R &= 0 \\ G_1 &= 0 \end{cases} \]

\[ \int \frac{1}{\text{loss less line}} = \text{dc} \]
Due to inherent properties of lossy transmission line, we assume that R, L, G, and C are effectively distributed along the entire length of the line.

2. Due to lossy nature of the line and due to limited conductivity of the line, some losses occur in the TL due to current flow along the line. The resistor R is responsible for total power dissipation taking place due to lossy nature of the line.

3. Due to current flow, due to magnetic fields, some magnetic energy will exist.

   The inductor L is responsible for total magnetic field in the TL.

4. Due to potential difference between the two lines, some electrical fields exist; therefore, some electrical energy is limited in the TL. The capacitor C is responsible for the total electrical energy stored in the transmission line.

5. The medium of dielectric between the two lines is in general lossy nature. Some power dissipation takes place as the current travels through the lossy dielectric.

   G is responsible for total power dissipation taking place due to lossy nature of the dielectric between the TL.
for a lossless line, $R$ and $Q$ are zero

as the voltage or current waveform are

outset on the TL, the wave is attenuated

exponentially and therefore the magnitude of

the voltage & current will decrease as

the wave propagates along the line.

Behavior of $V$ & $I$ along the line:

$I$ \[ \rightarrow \]

\[ \rightarrow \rightarrow \]

\[ - \rightarrow Z \]

\[ V = V^+ e^{-\sqrt{Z}} + V^- e^{\sqrt{Z}}, \]

propagating

along +Z

Incident wave

propagating

along -Z

Reflected wave

$V^+$ ---- Amplitude of voltage wave

propagating along +Z direction.

$V^-$ ---- Amplitude of wave propagating

along -Z

$\gamma$ ---- Propagation Constant

(Complex)

$\gamma = \alpha + j\beta$;

$\alpha$ ---- Attenuation Constant (nepers/m)

$\beta$ ---- Phase Constant ($\cdot$ d/m)
\[ I = \frac{1}{Z_0} \left( V^+ e^{-V^2} - V^- e^{+V^2} \right) \]

incident wave \[\rightarrow\] reflected wave

due to reverse direction of current

\[ Z_0 \] Characteristic Impedance.

\[ Z_0 \gamma (= \alpha, \beta) \]
--- Secondary Const of the line.

\[ Z_0 = \sqrt{\frac{R + j\omega L}{G_1 + j\omega C}} \]

\[ \gamma = \frac{1}{\sqrt{(R + j\omega L)(G_1 + j\omega C)}} \]

\[ = \alpha + j \beta \]

Basic features:

1. As the voltage or the current wave becomes impressed, for a lossy line it is subjective to attenuation as well as phase change.

Therefore the magnitude of voltage & current will decrease exponentially as the waves travel along the line.

If \( \alpha = 0 \) the wave propagation takes place without any attenuation.

Therefore the magnitude of voltage & current will remain constant at all the points along the line.

2. If \( \beta = 0 \) there is propagation & wave is entirely attenuate.
(a) The chart of the line represents the ratio of voltage to current at any point of an infinite long line.

(b) The propagation constant $\gamma$ of the characteristic impedance $Z_0$ are represented by the secondary constant of the line. Since the dependence upon the primary constant of $R, L, C + \phi$ of the line.

**Lossless line:**

$$R = 0 \quad G_1 = 0$$

$$Z_0 = \frac{\sqrt{R+j\omega L}}{j(\omega C)} = \frac{1}{\sqrt{L/C}} \quad \text{real; const.} \quad (\neq f(\omega))$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= j\omega \sqrt{LC} = \alpha + j\beta$$

$$\alpha = 0 \quad \beta = \omega \sqrt{LC}$$

$$\gamma = j\beta$$

$$\phi = \omega \sqrt{LC}$$

$$\frac{\phi}{\theta_p} = \frac{2\pi}{n}$$

$$\theta_p = \frac{1}{j\sqrt{LC}} \quad \text{const.}$$
**Summary:**

**Lossy Line**

\[ V = V^+e^{-j\beta_2} + V^-e^{+j\beta_2} \]
\[ I = \frac{1}{Z_0}(V^+e^{-j\beta_2} - V^-e^{+j\beta_2}) \]

\[ Z_0 = \sqrt{\frac{R+j\omega L}{G_1+j\omega C}} \]
\[ \gamma = \sqrt{(R+j\omega L)(G_1+j\omega C)} \]

R, L, G, C - primary const.
\( V(\omega) \); \( Z_0 \) - sec. const.

**Lossless Line**

\[ R = G_1 = 0 \]
\[ \alpha = 0 \]
\[ \gamma = j\beta \]
\[ \beta = \omega \sqrt{LC} \]
\[ \phi = \frac{\omega}{V_p} = \frac{2\pi}{\alpha} \]

\[ V_p = \frac{1}{\sqrt{LC}} \]

\[ Z_0 = \sqrt{\frac{1}{\omega C}} \]

\[ V = V^+e^{-j\beta_2} + V^-e^{+j\beta_2} \]
\[ I = \frac{1}{Z_0}(V^+e^{-j\beta_2} - V^-e^{+j\beta_2}) \]
Terminated T-L

\[ V_{max} \]
\[ V_{min} \]

Location of \( V_{1} \) voltage minima.

\[ Z_0 \]
\[ Z_L \]

\[ \frac{Z_0}{Z_L} = 1 \]

Source end

\[ V_S; I_S \]

Load end

\[ V_L; I_L \]

Case 1: \( Z_L \neq Z_0 \)

Mismatched line,

1. Max. power is not transfer to the load.
2. Incident & Reflected waves will exists.
3. Standing wave pattern will exists along the line.
   These for there is a stand form of Standing wave pattern.
4. Coefficient of Reflection has a finite value.
5. due to \( V_{max} & V_{min} \) Voltage Standing wave Ratio is finite.

\[ VSWR = S = \frac{V_{max}}{V_{min}} \]
Case 2: \( Z_L = Z_0 \)

- Matched line.

1. Maximum power is transferred from source to load.
2. Reflection coefficient is zero.
3. There is no reflected waves, no standing wave pattern, \( V_{max} = V_{min} \) & there by the voltage along the line, is const. at all the points.
4. The VSWR has a min. value of unity.

\[
S = \frac{V_{max}}{V_{min}} = 1
\]

Since \( V_{max} = V_{min} \)

In brief:

1. Reflection coefficient: \( \Gamma = \frac{V^-}{V^+} \)
   \[ |\Gamma| e^{j\phi} = \rho e^{j\phi} \]
2. Transmission coefficient: \( T = \frac{V_L}{V^+} \)
3. VSWR (S) = \( \frac{V_{max}}{V_{min}} \)
4. \( Z_{in} = \frac{V_s}{I_s} = \frac{V}{I} \mid_{-3} = 1 \)

Assume: Line is lossless

1. \( V = V^+ e^{-j\beta_2} + V^- e^{+j\beta_2} \)
2. \( I = \frac{1}{Z_0} \left( V^+ e^{-j\beta_2} - V^- e^{+j\beta_2} \right) \)
\[ Z = 0 \]
\[ V_i = V^+ + V^- \quad (1) \]
\[ I_i = \frac{1}{Z_0} (V^+ - V^-) \quad (2) \]
\[ \frac{V_i}{I_i} = Z_L = Z_0 \frac{V^+ + V^-}{V^+ - V^-} \]

\[
\Gamma = \frac{V^-}{V^+} = \frac{Z_L - Z_0}{Z_L + Z_0}
\]

If \( \frac{Z_L}{Z_0} = Z_L \)—— Normalized load impedance.

\[
\Gamma = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1} = \frac{Z_L - 1}{Z_L + 1}
\]

Transmission coefficient:

\[ T = V_L/V^+ \]

From equation (1)

\[ \frac{V_L}{V^+} = 1 + \frac{V^-}{V^+} \]

\[ T = 1 + \Gamma = 1 + \frac{Z_L - Z_0}{Z_L + Z_0} \]

\[ T = \frac{2Z_L}{Z_L + Z_0} = \frac{2Z_L}{Z_L + 1} \]

\[ S = \frac{V_{\text{max}}}{V_{\text{min}}} = \frac{|V^+| + |V^-|}{|V^+| - |V^-|} \]

\[ = \frac{1 + |V^+|/|V^-|}{1 - |V^-|/|V^+|} = \frac{1 + |M|}{1 - |M|} \]
\[ G = \frac{1+e}{1-e} \]

\[ c_{min} = 0 \quad \text{and} \quad s_{max} = \infty \]

\[ e_{max} = 1 \]

\[ Z_{in} = \frac{V}{I} \left| \frac{V + e^{-j\beta z} + V - e^{+j\beta z}}{V + e^{-j\beta z} - V - e^{+j\beta z}} \right| = \frac{Z_o}{-3} = 1 \]

\[ e^{j\beta l} = \cos \beta l + j \sin \beta l \]

\[ \frac{V^-}{V^+} = \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \]

\[ Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \]

**Summary:**

\[ \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_L - 1}{Z_L + 1} \]

\[ Z_L = \frac{Z_0}{1 - \Gamma} \]

\[ \Gamma = 1 + \frac{\alpha}{\beta} = \frac{2Z_L}{Z_L + Z_0} = \frac{Z_L}{Z_L + 1} \]

\[ \Gamma = \frac{1+e}{1-e} \quad \text{and} \quad \rho = \frac{S-1}{S+1} \]

\[ 0 \leq \rho \leq 1 \quad \text{and} \quad 1 \leq S \leq \infty \]

\[ Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \]
A lossless line of length \( l \) and characteristic impedance \( Z_0 \) is terminated by a load impedance \( Z_L \). Calculate:

1. Input impedance.
2. Reflection coefficient.
3. VSWR when:

\[
Z_L = 0 \quad \text{--- BC line}
\]
\[
Z_L = \infty \quad \text{--- OC line}
\]
\[
Z_L = Z_0 \quad \text{--- Matched line}
\]
\[
Z_L = jX \quad \text{--- purely reactive load.}
\]

**Case 1**

\[
Z_L = 0
\]

--- BC line

\[
Z_{in} = Z_0 \cdot \frac{Z_0 + jZ_0 \tan \beta l}{Z_0 + jZ_0 \tan \beta l}
\]

\[
= jZ_0 \tan \beta l
\]

--- purely reactive
--- S.C. stub line

\[
Z_{in} = Z_{sc} = jZ_0 \tan \beta l
\]

**Reflection Coefficient**

\[
\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}
\]

\[
\Gamma = \frac{Z_0^2 - Z_0}{Z_0^2 + Z_0} = -1 = \frac{V_-}{V_+}
\]

\[
\Gamma = e^{j\phi} = -1
\]

\[
e^{j\phi} = e^{-j\pi}
\]

\[
\phi = \frac{\pi}{2}
\]

\[
\phi = \frac{\pi}{2}
\]

\[
\{ e^{j\phi} = e^{-j\pi} = \cos \phi - j \sin \phi \} \implies \phi = 0
\]
Minima is located at the load end.

\[ s = \frac{1 + e}{1 - e} = \infty = \frac{V_{\text{max}}}{V_{\text{min}}} \]

Features:

1. The input amp. is purely reactive in nature. For the shortest length of the line this amp. is inductive nature.
2. The stub line is a portion of line which has been &c at the load end & has purely reactive s/p amp.
3. A short circuit stub can be used for matching transmission line with the load amp. for max. power transfer.
4. The reflective voltage & incident voltage are equal in magnitude but are phase shifted by 180°.
5. The voltage minima occurs at the load end & the 1st voltage maxima occurs at a distance of \( \frac{\lambda}{4} \) from the load end.
Case: 2

\[ Z_L = \infty \]

\[ Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta L}{Z_0 + jZ_L \tan \beta L} \]

\[ = Z_0 \frac{1 + j\left(\frac{Z_0}{Z_L}\right) \tan \beta L}{\left(\frac{Z_0}{Z_L}\right) + j \tan \beta L} \]

\[ Z_{oc} = -jZ_0 \cot \beta L \]

--- purely reactive. Oc stub line.

\[ Z_{sc} = jZ_0 \tan \beta L \]

\[ Z_{oc} = -jZ_0 \cot \beta L \]

\[ Z_{sc} \cdot Z_{oc} = Z_0^2 \]

\[ \rightarrow Z_0 = \sqrt{Z_{sc} \cdot Z_{oc}} \]

\[ \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = 1 - \frac{Z_0 / Z_L^{\infty}}{1 + Z_0 / Z_L^{\infty}} = 1 \]

\[ \Gamma = \rho e^{j \theta} = 1 \]

\[ \rho = 1 \]

\[ \theta = 0 \]

\[ S = \frac{1 + \rho}{1 - \rho} = \infty = \frac{V_{\text{max}}}{V_{\text{min}}} \]

--- \( V_{\text{max}} \) occurs at load end.
O.C. Line:

\[ Z_L = \infty \]
\[ r = 1 \]
\[ \epsilon = 1 \]
\[ \phi = 0^\circ \]
\[ S = \infty \]
\[ Z_{oc} = -jZ_{oc\text{capacitive}} \]

Features:

1. The input amp. is purely reactive & for a shortest length of the line it's capacitive in nature.
2. The o.c. & stub line can be used to matched any transmission line with the load amp. for max. power transfer.
3. The reflected & incident voltages has same magnitude and are in-phase.
4. Voltage maxima occurs at the load end.
5. When the line is first s.c. & then o.c.
6. The voltage minima shifted by distance \( \frac{\lambda}{4} \) from the load end towards a source end.
7. The characteristic amp. of the line is a geometric mean of input amp. of the line when it's is s.c. & then o.c.
8. An impedance inversion takes place when the line is first s.c. & then o.c. vice-versa.
Therefore Inductive amp. is transformed into capacitive Inductive & vice versa.

case 3: 
\[ Z_L = Z_o \]

Matched line
\[ \Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = 0 = \frac{V_-}{V_+} \]

\[ \Gamma = e^{j\theta} = 0 \]

\[ S = 1 \]
\[ V_{max} = V_{min} \]

\[ Z_{in} = Z_o \]

\[ Z_{in} = Z_o \]

\[ V_{max} = V_{min} \]
1. A perfectly matched line behaves as infinitely long line since in each case the electrical length of the line is equal to the electrical length of the line.

2. There is no reflected waves & reflection coefficient is zero, VSWR has min. value of unity.

3. Maximum power is transferred to the load & so that $V_{max} = V_{min}$ there is no standing waves & there box the voltage along the line is uniform at all the points.

**Case: 4**

$$Z_L = jx$$

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

$$Z_{in} = j \left[ \frac{Z_0 \frac{x + Z_0 \tan \beta l}{Z_0 - x \tan \beta l}}{Z_0 - jx \tan \beta l} \right]$$

- Real
- Pure reactance

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma = \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right| e^{j\theta}$$

$$s = \frac{1 + \frac{V_{max}}{V_{min}}}{1 - \frac{V_{max}}{V_{min}}}$$

$$Z_L = \frac{1}{jwL}$$

$$Z_L = \frac{1}{jwC}$$

High

Low
\[ \theta = -2 \tan^{-1}\left(\frac{X}{Z_0}\right) \]  
(22)

Features:
1. If the line is terminated by a purely reactive load then s/l amp. is also purely reactive.
2. The location of voltage max. & v/ min. on the standing wave pattern will depend upon the type of the reactive load.
   - For inductive load the v/ max will occur at the load end whereas as minima will occur at the load end if it is capacitive nature.

Ex: A lossless TL of length \( L \) has characteristic Imp. of \( Z_0 \). \& is terminated by load Imp. \( Z_L \).

Find input Imp. of the line when:\n
Case 1: \( L = \pi \)
Case 2: \( L = \pi/2 \)
Case 3: \( L = \pi/4 \) ---- QWT
Case 4: \( L = \pi/8 \) Quarter wave transformer

To find:\n\[ Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta L}{Z_0 + jZ_L \tan \beta L} \]
\[
\tan \beta_1 = \tan \left( \frac{2\pi}{\lambda} \cdot \frac{d}{2} \right) = 0
\]

\[
Z_{in} = Z_0 \frac{Z_L + 0}{Z_0 + 0}
\]

\[
Z_{in} = Z_L
\]

**Case 2:** \( d = d/2 \)

\[
\tan \beta_1 = \tan \left( \frac{2\pi}{\lambda} \cdot \frac{d}{2} \right) = 0
\]

\[
Z_{in} = Z_L
\]

**Case 3:** \( d = d/4 \)

\[
\text{Quarter wave transformer}
\]

\[
\tan \beta_1 = \tan \left( \frac{2\pi}{\lambda} \cdot \frac{d}{4} \right) = \infty
\]

\[
Z_{in} = Z_0 \left( \frac{Z_L / \tan \beta_1 + jZ_0}{Z_0 / \tan \beta_1 + jZ_L} \right)
\]

\[
Z_{in} = \frac{Z_0}{Z_L}
\]

\[
Z_0 = \sqrt{Z_{in} \cdot Z_L}
\]

\[
\begin{array}{c}
\text{Zin} \\
\text{Z_0} \\
\text{Z_L}
\end{array}
\]

\[
\leftarrow \text{d/4} \rightarrow
\]
1. A QWT exists an impedance universal. Therefore if the load \( Z_L \) is inductive then the S/P impedance is capacitive & vice-versa.

2. A \( A/A \) section of the line matched two amp. \( Z_L \) & \( Z_{in} \), a perfect matching take place whenever the load \( Z_L \) & S/P amp. is purely resistive amp.

3. A QWT section of the line is used to transform given load amp. \( Z_L \) to the desired S/P amp. using a QWT whose characteristic amp. is the geometric mean of the load \( Z_L \) & amp. input amp. \( Z_L \).

Case 4: \( \lambda = \frac{\pi}{8} \)

\[
\tan \beta L = \tan \left( \frac{2\pi}{\lambda} \cdot \frac{\lambda}{8} \right) = 1
\]

\[
Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta L}{Z_0 + jZ_L \tan \beta L} = 1
\]

\[
Z_{in} = Z_0 \frac{Z_L + jZ_0}{Z_0 + jZ_L}
\]

--- Complete nature

\[ |Z_{in}| = Z_0 \]

1. For \( A/A \) section of the line the output impedance is always complex on nature irrespective of the nature of load amp. \( Z_L \).

2. The magnitude of output impedance of \( A/A \) section of the line is always as numerically equal to the characteristic of the line.
Find the chity impedance $Z_{01}$ oh the 1st line so that there is no reflected wave on it.

$$Z_{01} = Z_{LL} = Z_{im}^2 \parallel Z_{im}^3$$

matched line

$$= \frac{Z_{01} \parallel Z_{03}}{}$$

matched line

$$= 200 \parallel 300$$

$$= 120 \Omega$$

$$Z_{ad} = Z_{im} = ?$$

$$Z_0 = 200 \Omega$$

to find:

$$Z_{im}$$

$$Z_{im} = \frac{Z_{ad} \parallel Z_{ad}}{}$$

$$= \frac{j100 \tan \beta L}{Z_0}$$

$$= j100 \tan \left( \frac{2\pi \cdot 2/16}{\lambda} \right)$$

$$Z_{m} = \frac{j100 \times j200}{j100 + j200} = -100 \Omega$$

$$Z_{m} = \frac{j100 \times j200}{j100 + j200} = \frac{j200}{3} \Omega$$
\[
Z_L3 = Z_{\text{in}1} \parallel Z_{\text{in}2}
\]
\[
= \frac{Z_0^2}{Z_L1} \parallel \frac{Z_0^2}{Z_L2} \geq \frac{2500}{100} \parallel \frac{2500}{200}
\]
\[
\geq \frac{25}{1} \parallel \frac{25}{2} \geq \frac{25}{3} \Omega
\]

\[
\Gamma = \frac{Z_{L3} - Z_0}{Z_{L3} + Z_0} = \frac{25/3 - 50}{25/3 + 50} = -\frac{5}{7}
\]

\[
Z_{\text{in}} = \frac{Z_0^2}{Z_{L3}} = \frac{2500}{25/3} = 300 \Omega
\]

\[
\Gamma = \frac{Z_{\text{in}3} - Z_0}{Z_{\text{in}3} + Z_0} = \frac{300 - 50}{300 + 50} = +\frac{5}{7}
\]

Ans.
\[ Z_{01} = 30j 2 \]
\[ Z_{02} = 30 \]
\[ Z_{03} = 60 \]

To find: \[ S \text{ on } Z_{03} \text{ line} \]

\[ S = \frac{1+e}{1-e} \]

\[ e = 1 \Gamma \]

\[ \begin{align*}
\Gamma' &= \frac{Z_{L3} - Z_{03}}{Z_{L3} + Z_{03}} \\
Z_{L3} &= Z_{in} = Z_{1/8} + Z_{in 1} \\
&= jZ_{02} \frac{\tan \beta l}{\tan \left( \frac{2\pi}{A} \cdot \frac{1}{8} \right)} \approx 1 \\
&= j30
\end{align*} \]

\[ Z_{L3} = j30 + 60 \]

\[ \Gamma = \frac{Z_{L3} - Z_{03}}{Z_{L3} + Z_{03}} = \frac{60 + j30 - 60}{60 + j30 + 60} = \frac{j30}{120+j30} \]

\[ = \frac{j1}{4+j1} \]

\[ \text{bind } \Gamma = 1 \Gamma' \]

\[ \text{bind } S = \frac{1+e}{1-e} \]

Ex:

\[ \Gamma_3 = ? \]

\[ \Gamma = 0.6 \text{ } e^{-j30} \]

\[ Z_0 \]

\[ Z_L \]

\[ 0.1 \text{ } \Omega \]

\[ 7 \text{ } \Omega \]

\[ -3 = \text{ source end} \]

\[ 3 = \text{ load end} \]
\[ \Gamma_s = \rho_s e^{j\theta} \]
\[ = 0.6 e^{j0} \]

\[ \theta = \phi + (-30^\circ) \]

due to load

due to path difference

\[ \phi = (\text{path diff.}) \times \frac{\frac{2\pi}{\lambda}}{\beta} \]

\[ = 2 \times \frac{\frac{2\pi}{\lambda}}{\beta} \]

\[ = -2 \times 0.1 \times 2\pi \]

\[ = -0.4 \pi \]

\[ = -0.4 \times 180^\circ \]

\[ \phi = -72^\circ \]

\[ \theta = \phi_1 + (-30^\circ) \]

\[ \theta = -72^\circ - 30^\circ \]

\[ \theta = -102^\circ \]

\[ \Gamma_S = \rho_s e^{j\theta} \]

\[ = 0.6 e^{-j102^\circ} \]

\[ \Gamma_S = 0.6 e^{j258^\circ} \]

Ans.
Distortion less line :-

\[ y = a + ib \]

\[ y = \frac{1}{(R+j\omega L)(G_1+j\omega C)} \]

\[ a = f(w, w^2, w^3) \]

\[ b = f(w, w^2, w^3) \]

Frequency distortion -

\[ a = f(w, w^2, w^3) \]

Causes freq. distortion.

\[ a \neq f(w) \]

To avoid freq. distortion.

\[ \text{equalizes lattice network.} \]

1. The frequency distortion occurs since various frequency components are subjective to different amount of attenuation.

   This changes the quality of the voice signal.

2. To avoid freq. distortion all the frequency components must be subjected to same amount of attenuation so that quality of voice remains same.

   Therefore freq. response attenuation constant \( \alpha \) should be constant & independent
practically the only distortion is avoided by using an equaliser which represents a lattice.

\[ \beta = f(w, w^2, w^3) \]

Causes phase and delay distortion.

\[ \beta = \frac{W}{V_p} \] must be constant to avoid dispersion.

\[ \beta \propto w \]

\[ \beta = kW \]

**1.** Various frequency components are subjective to shift amount of phase shift which causes the phase and delay distortion.

To avoid the phase or delay distortion phase const. \( \beta \) must be directly proportional to \( v_p \) of operation so that the phase velocity along the line must remain constant. Therefore, the frequency response of the phase const. \( \beta \) must have linear variation with the same frequency variation.

**Practical Path Distortion is Avoided by using phase or delay equaliser.**
for distortion less line

\[
\begin{align*}
\alpha & \neq f(w) \\
\beta & = Kw
\end{align*}
\]

- to avoid freq. distortion.
- to avoid phase of delay distortion.

\[
\Rightarrow \quad RC = LG
\]

condition for a distortion less line.

**Characterstic Impedance of distortion less line**

\[
Z_0 = \sqrt{\frac{R+jwL}{G_1+jwC}} \quad \text{and} \quad RC = LG
\]

\[
R = \frac{Lm}{C}
\]

\[
\Rightarrow Z_0 = \sqrt{\frac{LG/C + jwL}{G_1 + jwC}} = \sqrt{\frac{L(G_1 + jwC)}{C(G_1 + jwC)}}
\]

\[
Z_0 = \sqrt{\frac{L}{C}} \quad \text{Same as that of lossless line}
\]

**Case 1:** at high frequency

\[
\omega L \gg R \quad \omega C \gg G_1
\]

\[
Z_0 = \sqrt{\frac{R^2 + jwL}{G_1}} \approx \sqrt{\frac{1}{C}}
\]

**Case 2:** at low frequency.

\[
\omega L < R \quad \omega C < G_1
\]
(2) The characteristic gain of a lossless & distortionless line is same & depends only upon the primary constant L & C.

(3) At high frequency, the TL will always behave as a lossless as well as distortionless line irrespective of the condition $RC = LG_1$ is satisfied or not.

(4) At low frequency the line in general does not behaves as a distortionless line unless the condition $RC = LG_1$ is satisfied.

Summary:

<table>
<thead>
<tr>
<th>Distortionless Line</th>
<th>Lossless line</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x \neq f(w)$</td>
<td>$R = G_1 = 0$</td>
</tr>
<tr>
<td>$\beta = K\omega$</td>
<td>$x = 0$</td>
</tr>
<tr>
<td>$\beta = \omega$</td>
<td>$\beta = \frac{\omega}{v_p}$</td>
</tr>
<tr>
<td>$v_p = \frac{1}{\sqrt{LC}}$</td>
<td>$v_p = \frac{1}{\sqrt{LC}}$</td>
</tr>
<tr>
<td>$z_0 = \sqrt{\frac{L}{C}}$</td>
<td>$z_0 = \sqrt{\frac{L}{C}}$</td>
</tr>
<tr>
<td>$R \cdot C = LG_1$</td>
<td>$R \cdot C = LG_1$</td>
</tr>
</tbody>
</table>
Consultation: 1) For a distortion less line, it must be independent of freq. but may have a finite value.

Therefor on general a distortion line is a lossless line.

2) For a lossless line, \( R = G = 0 \) are zero if there is no the condition \( RC = LG \) is always satisfied.
Since \( \alpha = 0 \) for such line it's independent of freq. \( \omega \).

therefor any loss less line is always a distortion less line.

3) Practically any transmission line is always a lossy line & these both cannot be a distortion less line.
Such line may be made distortion less by suitably selecting the material such that the condition \( RC = LG \) is always satisfied.

\[ Z_0 = 50 \, \Omega \]
\[ R = 0.1 \, \Omega /m \]

\[ Z_0 = \sqrt{\frac{L}{C}} \]
\[ \frac{L}{C} = Z_0^2 = 2500 \]

\[ RC = LG \Rightarrow G = R \cdot \frac{C}{L} = \frac{0.1}{2500} \, \text{S/m} \]

To find \( \alpha \) for distortionless line.
\[ V = v^+ e^{-j\beta l} + v^- e^{j\beta l} \]

\[ I = \frac{1}{Z_0} \left( v^+ e^{-j\beta l} - v^- e^{j\beta l} \right) \] (35)

\[ V^+_l \left|_{l=1} \right. = V_l = \frac{v^+ e^{j\beta l} + v^- e^{-j\beta l}}{e^{j\beta l} = \cos \beta l + j \sin \beta l} \]

\[ \frac{v^-}{v^+} = \Gamma = \frac{Z_l - Z_0}{Z_l + Z_0} \quad ; \quad Z_l = \frac{V_l}{I_l} \]

\[ V^+ \Rightarrow \frac{V_l}{V^+} = T \quad ; \quad V^+ = \frac{V_l}{T} \]

\[ T = \frac{Z_l}{Z_l + Z_0} \quad ; \quad Z_l = \frac{V_l}{I_l} \]

\[
\begin{bmatrix}
V_s \\
I_s
\end{bmatrix} =
\begin{bmatrix}
\cos \beta l & j \sin \beta l \\
(j/Z_0) \sin \beta l & \cos \beta l
\end{bmatrix}
\begin{bmatrix}
V_l \\
I_l
\end{bmatrix}
\]

\[
\begin{bmatrix}
V_l \\
I_l
\end{bmatrix} =
\begin{bmatrix}
\cos \beta l & -j Z_0 \sin \beta l \\
(-j/Z_0) \sin \beta l & \cos \beta l
\end{bmatrix}
\begin{bmatrix}
V_s \\
I_s
\end{bmatrix}
\]

**Example:**

\[ R_s = 10 \Omega \]

\[ Z_m = 50 \Omega \]

\[ R_L = 20 \Omega \]

\[ 10V = V_L \]

\[ \text{To find:} \quad \{ V_s; I_s; I_l; Z_m; V_3 \} \]
\[ \alpha = f(w) \]
\[ \gamma = \sqrt{(R + j\omega L)(G_1 + j\omega C)} = \alpha + j\beta \]
\[ \alpha = f(w) \]
\[ \gamma = \alpha + j\beta = \sqrt{RG_1} \]
\[ \alpha = \sqrt{RG_1} = \sqrt{0.1 \times 0.1} \]
\[ \alpha = 0.002 \ \text{ nepers/m} \]
\[ 1 \ \text{nepers} = 6.686 \ \text{dB} \]
\[ \alpha = 0.002 \times 6.686 \]
\[ \alpha = 0.013 \ \text{dB} \]

**Transmission Matrices:**

\[ Z_0 \]
\[ Z_L \]

\[ V_s; I_s \]
\[ V_L; I_L \]

\[ Z = 0 \]

Source end
\[ Z = 0 \]

Load end

\[ V_s \]
\[ I_s \]

\[ V_L \]
\[ I_L \]

For a lossless line.
\[ I_L = \frac{V_l}{R_L} = \frac{10}{20} \]

\[ I_L = \frac{1}{2} \text{ Amp.} \]

\[ V_8 = \frac{\cos \beta l \cdot V_i - j \cdot Z_0 \cdot \sin \beta l \cdot I_L}{\cos \left( \frac{\sqrt{2} \pi}{\lambda} \right)} \]

\[ V_8 \sin \left( \frac{\sqrt{2} \pi}{\lambda} \right) = 0 \]

\[ V_8 = -10V \]

**Simple method for \( I_S \)**

\[ Z_{in} = Z \bigg|_{\lambda/2} = Z_L = 20 \Omega \]

\[ Z_{in} = 20 \Omega \]

**Source Circuit Diagram**

\[ I_S = \frac{V_8}{Z_{in}} = \frac{-10}{20} = -\frac{1}{2} \text{ Amp} \]

\[ V_S = I_S (10 + 20) \]

\[ V_S = -\frac{1}{2} \left( 10 + 20 \right) \]

\[ V_S = -15 V \]
Stub Matching:
Required when \( Z_L \neq Z_0 \)

Case 1:
Short circuit shunt stub

\[ d = \text{location of stub} \]
\[ l = \text{length of stub} \]

\[ S_{\text{main line}} = 1 \]
\[ S_{\text{load line}} = \infty \]

\[ \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \]
\[ \epsilon = |\Gamma| \]
\[ S = \frac{1 + \epsilon}{1 - \epsilon} \]

To find:
- \( d \) and \( l \)

1. \( Z_L = \) given
2. \( \frac{Z_L}{Z_0} \) \( \equiv Z_L \)
3. \( \bar{Z}_L = \frac{1}{Z_L} \)
(4) Move a distance 'd' such that

\[ Y_{aa'} = 1 + jB \]

Comments : The location of the stub will adjust their normalised value of Real part of admittance at 'aa' becomes 0.

\[ Y_{aa'} = \frac{Y_{aa'} - jB}{1 + jB} \]

(5) Adjust the length l of the stub so that its normalised admittance at e aa is equal to opposite to get an admittance part of

\[ Y_{bb} = 1 + jB + \left( \frac{\sqrt{B}}{1 + jB} \right) = 1 \]

(6) \[ Z_{bb} = \frac{1}{Y_{bb}} = 1 \]

(7) \[ \frac{Z_{bb}}{Z_0} = 1 \]

\[ Z_{bb} = Z_0 \]

Hence the TL is perfectly matched with the effectively load amp. at 'bb'.

[10] Hence for to the left side of bb' the line is perfectly matched, there is no reflected waves, no standing wave pattern, reflection coefficient is zero. Therefore has a
Minimum value of \( \text{USWR} \) is unity. Therefore max. power is transferred from
the source to load.

The stub line will act as only the reactive power since the dip amp. of stub is purely
reactive.

This reactive power is not a useful power.

The entire real power or the useful power is transmitted to the load impedance \( Z_L \).

Hence max. real power is been transferred to load impedance \( Z_L \).

\[ Z_L = (100+j300) \Omega \]
\[ Z_0 = 100 \Omega \]

Real part

Both are equal then normalized is equal to

\[ d = 0 \]

Stub is connected at the load.

\[ Z_L = 200+j300 \]
\[ Z_0 = 100 \]

\[ d \neq 0 \]

Stub is connected at some specific distance from the load.

1. The stub Matching is used only by a short circuit or open circuit stub line.

Discrete components of L & C are not used for stub matching.
Case 2: Open circuit shunt stub:

\[ Z_0 \]

\[ Z_L \neq Z_0 \]

Case 3: SC series stub:

\[ Z_0 \]

\[ Z_L \neq Z_0 \]
(1) The S.C. stub is always preferred since the adjustment of the length is more conventional practically.
(2) The A.C. stub is normally not preferred since the adjustment of the length is practically not convenient.
(3) An A.C. stub x has a retina and an e.m. power is radiated from it.
(4) The stub stub is always preferred since the main line remains unaffected when the load is varied over a wide range.
(5) The series stub is never preferred since the main line is affected if the load Zl is varied over a wide range.

Therefore for variable load the S-C. stub stub matching is the best whereas A-C. stub matching is the worst.
Double stub matching is generally preferred over the single stub matching because of more flexibility in the variation of length of each stub \( l_1 \) & \( l_2 \).

Using double stub matching we are not able to match all the type of load with the characteristic impedance of the line.
Variation of impedance along the line

\[
Z_{\text{max}} = \frac{V_{\text{max}}}{I_{\text{min}}} = \frac{V_{\text{max}}}{V_{\text{min}}/Z_0} = Z_0 \cdot S
\]

\[
\left(\frac{Z_{\text{max}}}{Z_0}\right) = S
\]

\[
Z_{\text{min}} = \frac{V_{\text{min}}}{I_{\text{max}}} = \frac{V_{\text{min}}}{V_{\text{max}}/Z_0} = \frac{Z_0}{S}
\]

\[
\left(\frac{Z_{\text{min}}}{Z_0}\right) = \frac{1}{S}
\]

\[
\frac{1}{S} \leq Z \leq S
\]

**A. Transient Response in T.L.:**

- Line is mismatched at the source end. \( R_s = 20 \Omega \)
- Line is mismatched at the load end (\( Z_L \neq Z_0 \))

\[
Z_L = 200 \Omega
\]

\[
Z = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{200 - 100}{200 + 100} = \frac{1}{3}
\]

**Assumption:**

1. Line is lossless.
2. \( L \) is large
3. \( T = \frac{d}{v_p} \) time taken by voltage waveform to reach from source to load end.

\[
T = \frac{2100 \text{ mSec}}{1000} = 2.1 \text{ mSec}
\]
\[ P_s = \frac{R_s - z_0}{R_s + z_0} = \frac{50 - 100}{50 + 100} = \frac{1}{3} \]

To find:

\[ V(t) \propto t \]

- **transient response of the line.**

\[ V_s = \frac{100}{50 + 100} = \frac{2}{3} \times 12 = 8 \text{ V} \]

**Source diagram:**

- \( T_s = \frac{1}{3} \)
- \( T = \frac{1}{3} \)
- \( 2T = \frac{2}{3} \)
- \( 3T = \frac{3}{3} \)
- \( 4T = \frac{4}{3} \)
- \( 5T = \frac{5}{3} \)

**Load diagram:**

- \( V(t) \)
- \( 8 \text{ V} \)
- \( T = 8/3 \)
- \( 2T = -8/3 \)
- \( 3T = -8/3 \)
- \( 4T = 8/3 \)
- \( 5T = 8/3 \)
Sources of $E$ (Electrical Field):

1. **Point Charge**:
   \[ q \cdot \frac{1}{r^2} \cdot \hat{r} = \text{Coulomb's Law} \]

2. **Line Charge**:
   \[ dl = e_l \cdot dl \]
   \[ q = \int_C e_l dl \]
   line charge density
   \[ C \] charge per unit length ($C/m$)

3. **Surface Charge**:
   \[ d\sigma = e_s \cdot ds \]
   \[ q_s = \iint_S e_s \cdot ds \]
   surface charge density ($C/m^2$)

4. **Volume Charge**:
   \[ d\rho = \epsilon \cdot dv \]
   \[ q_v = \iiint_V \epsilon \cdot dv \]
   volume charge density ($C/m^3$)
A Source of \( \vec{B} \) (Mag. field)

**Biot-Savart's Law**

\[
\frac{dI}{d} = \int \vec{B} \times \frac{I}{l} \\
(A) \rightarrow I = I_0, \Rightarrow \vec{B} = \vec{B}_0 \text{ static field} \\
\Rightarrow I_0 \sin \omega t \\
\Rightarrow \vec{B} = \vec{B}_0 \sin \omega t \text{ time varying mag. field} \\
\]

**Current Element**

\[
\int d\vec{l} = \int dI \\
\]

**Convention:**

by convention the direction of current \( I \) and elementary length \( dl \) are taken same on all electromag. problems.

**Surface Current**

\[
I = \iint_{\partial S} \vec{J}_s \cdot d\vec{l} \\
\]

**Surface Current Density**

\( A/m \)

**Volume Current**

\[
I = \iiint_{V} \vec{J}_v \cdot d\vec{V} \\
\]

**Volume Current Density**

\( A/m^2 \)
Continuity eqn.

\[ I = - \frac{dQ}{dt} \]

Convection of charge

\[ \Delta t = 1 \text{ sec} \]

\[ \Delta t = 5 \mu \text{C} \]

\[ Q = 2 \mu \text{C} \]

\[ I = \frac{\Delta Q}{\Delta t} = \frac{Q_i - Q_f}{\Delta t} \]

\[ (2.5) \times 10^{-6} \]

\[ 1 \times 10^{-3} \]

\[ = 2 \text{ mA} \]

Conductivity (S/m)

\[ \nabla \cdot \vec{J} = \sigma \frac{\partial \vec{E}}{\partial t} \]

Volume charge density (C/m²)

\[ \text{Continuity eqn.} \]

Where

\[ \vec{J} = \sigma \vec{E} \]

Ohm's law

\[ \vec{J} \text{ is solenoidal in nature.} \]

KCL eqns.

**Statement:**

1. The diversion of volume current density \( \vec{J} \) at any point in the electromagnetic region is always equal to the rate of decrease of the volume charge density \( \vec{P} \) with respect to time \( t \).

2. Hence, there is a continuity between the decrease of volume charge density \( \vec{P} \) and the corresponding production of volume current density \( \vec{J} \).

3. In a charge free region, the diversion of volume current density \( \vec{P} \) at any point is always equal to zero.
and hence volume current density is always solenoidal and forms a closed loop.

\( \mathbf{J} = 0 \)

- on perfect conductor \((\sigma = \infty)\)
- on dielectric \((\sigma = 0)\)

1. The charge density \(\rho\) is always equal to zero on a perfect conductor or a perfect dielectric conductor.

2. Volume charge density is finite on a medium where the conductivity is finite.

The decay of volume charge density will depend upon the conductivity of region.

Higher is the conductivity higher is the rate decay of charge & vice versa.

Maxwell's eqns. in their general time-varying form:

Differential form (point form)

\[ \nabla \cdot \mathbf{D} = \rho \]

Integral form

\[ \oint \mathbf{B} \cdot d\mathbf{S} = \mathbf{M} = \int \int \int \mathbf{E} \cdot d\mathbf{V} \] Gauss's law for electric fields

Electric flux

\[ \nabla \cdot \mathbf{B} = 0 \]

\[ \oint \oint \mathbf{B} \cdot d\mathbf{S} = 0 \] Gauss's law for magnetic fields
\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]

\[ \vec{E} \cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \]

\[ \text{emf} \quad \text{Faraday's law of e.m. induction.} \]

\[ \oint_S (\vec{J}_c + \vec{J}_d) \cdot d\vec{s} \]

\[ \text{mmf} \quad \text{Modified Ampere's circuital law} \]

Eqn. using displacement current density concept

\[ \vec{J}_c = \sigma \vec{E} \quad \text{Cond. current density (A/m}^2\text{)} \]

\[ \vec{J}_d = \frac{\partial \vec{D}}{\partial t} \quad \text{displacement current density (A/m}^2\text{)} \]

**Statements:**

1. (i) The divergences of electrical flux density \( \vec{D} \) at any point in the electro mag. region is always equal to the volume charge density \( \rho \).

2. (b) The net electrical flux passing through any closed surface area \( S \) is always equal to total charge enclose within the surface. Area \( S \).

2(a) The divergences of mag. flux density \( \vec{B} \) is always equal to zero since reasons: (i) Mag. field \( \vec{B} \) line are always closed in nature.

(ii) The Mag. charges in the isolated room do not exists in nature.
2(b) The net mag. flux passing through any closed surface area S is always equal to zero.

3(a) The curl of electric field intensity \( \vec{E} \) at \( \vec{r} \) is always equal to the rate of decrease of mag. flux density \( \vec{B} \) w.r.t. time \( t \).

3(b) Net emf. produced is always equal to the surface integral of rate of decrease of mag. flux density \( \vec{B} \) w.r.t. time \( t \).

4(a) The curl of mag. field intensity \( \vec{H} \) is always equal to the sum of conduction current density \( \vec{J}_c \) & the displacement current density \( \vec{J}_d \).

4(b) Total mmf produced is always equal to the surface integral of the sum of conduction current density \( \vec{J}_c \) & displacement current density \( \vec{J}_d \).

Special Cases:

case 1: For static fields:

\[
\frac{d\vec{B}}{dt} = 0
\]
\[
\frac{d\vec{D}}{dt} = 0
\]
Elec 2:

- for perfect dielectric \( (\sigma = 0) \)
- or
- non-conducting medium
- or
- loss less medium
- or
- free space

\[
\vec{J}_c = \sigma \vec{E} = 0 \\
e = 0
\]

for good conductor, \( \sigma \) is high

\[
\vec{J}_e = 0 \\
e = 0
\]

for time-harmonically or sinusoidally varying fields:

\[
\begin{align*}
\vec{D} &= \vec{D}_0 e^{j\omega t} \\
\vec{B} &= \vec{B}_0 e^{j\omega t} \\
\omega &= j\omega \vec{B} \\
\Rightarrow \quad j\omega &= \omega j\vec{B}
\end{align*}
\]

\[
\Rightarrow \quad (j\omega)^2 = -\omega^2
\]

- \( \vec{D} = \vec{D}_0 e^{j\omega t} \)
- \( \nabla \cdot \vec{D} = \epsilon \)
- \( \vec{B} = \vec{B}_0 e^{j\omega t} \)
- \( \nabla \cdot \vec{B} = 0 \)
- \( \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -j\omega \mu \vec{H} \)
- \( \nabla \times \vec{H} = \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t} \)
  \[= \sigma \vec{E} + j\omega \epsilon \vec{E} = (\sigma + j\omega \epsilon) \vec{E} \]
\[ J_{\text{total}} = J_c + J_d \]
\[ = \sigma E + \frac{\delta \Phi}{\delta t} \]
\[ = \sigma E + j\omega \varepsilon E \quad \text{for general medium} \]
\[ = \sigma E \quad \text{for good conductor} \]
\[ = j\omega \varepsilon E \quad \text{for good dielectric} \]

\[ \tan \phi = \frac{|J_c|}{|J_d|} = \frac{|\sigma E|}{|j\omega \varepsilon E|} = \frac{\sigma \varepsilon}{\omega \mu} \]

\[ \tan \phi = \frac{\varepsilon}{\omega \mu} \gg 1 \quad \text{Good Conductor.} \]
\[ \ll 1 \quad \text{Good dielectric.} \]

\( \phi \) points:
1) For good conductors, the conductivity is high, thus the conduction current density is dominant.
2) For good dielectric, the conductivity is low, the conduction current density is negligible, and the displacement current density is more dominated.
3) The loss tangent is the ratio of the magnitude of conduction current density to displacement current density.

This is a major of total loss occurring in a material due to its conductivity as specified frequency.
4) Depending upon the frequency of operation, the medium may behave as a good conductor.
or good dielectric.

In general, my material may behave as a good conductor at low freq., whereas some material may behave as a good dielectric at high frequency.

**Conclusions**

Therefore, depending upon application, we operate a device at high freq., and low freq., so that it can operate at a good dielectric & good conductor.

\[ \vec{P} = \vec{E} \times \vec{H} \]

\[ \frac{\mu}{\varepsilon}, \frac{A}{m} \]

\[ W/m^2 \]

Power, density at a point

\[ \text{power} = \int \int \int \vec{P} \cdot d\vec{s} \]

If \( \vec{E} \perp \vec{H} \perp \vec{P} \)

Transverse, e.m. wave

TEM wave \( \rightarrow \) uniform plane wave (plane wave)

\[ \vec{P} = \vec{E} \times \vec{H} = EH \sin \phi, \phi_n \]

\[ \sin \phi = 1 \]

\[ |\vec{P}| = |\vec{P} = EH| \]

\[ = E_{mag} \cdot H_{mag} \]

\[ \frac{E}{H} = \eta \quad \text{Intrinsic impedance of the medium.} \]
\[ \frac{E}{H} = \eta = \frac{jwM}{\sqrt{\sigma + jw\varepsilon}} \]

For general lossy medium \((\mu, \varepsilon, \sigma)\)

For lossless medium \((\sigma = 0)\)

\[ \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} \]

\[ \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 120 \pi \approx 377 \Omega \]

For free space

\[ \sigma = 0 \]

\[ \mu = \mu_0 \]

\[ \varepsilon = \varepsilon_0 \]

\[ P = \frac{E \cdot H}{\eta} \]

\[ P = \frac{1}{2} \varepsilon \cdot H_m^2 \]

\[ P = \frac{1}{2} \eta \cdot H_m^2 \]

\[ P = \frac{1}{2} \frac{\varepsilon_0}{\eta} E_m^2 \]

\[ \text{Imp. Points:} \]

1. The projecting vector \(P\) represents power density at a point.

When integrate over any closed surface area, the total power blow on the specified direction.

To give the direction of propagation of EM waves \(E\) is always perpendicular to the plane made by \(E\) and \(H\) vectors.
for a transfer e.m. waves E, H, and P are mutually perpendicular to each other.

In any e.m. region the ratio of the E and H fields is always constant. It is represented by the intrinsic impedance of the medium.

This ratio depends only upon the const. of the medium.

For a free space this ratio has a universal constant value of 120π m or 377 Ω.

Ex:
An e.m. waves is travelling along -y direction. & has only x component of electric fields.

Find the magnetic field intensity associated with e.m. waves.

\[ \mathbf{p} = \mathbf{E} \times \mathbf{H} \]

\[ \mathbf{p} = -P_y \mathbf{a}_y \] direction of prop. of wave.

\[ \mathbf{E} = E_z \mathbf{a}_z \]

To find:
\[ \mathbf{H} = ? \]

\[ -\mathbf{a}_y \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ a_y & a_x & a_z \\ \end{vmatrix} = +\mathbf{a}_x \]

\[ \mathbf{p} = \mathbf{E} \times \mathbf{H} \]

\[ (P_y \mathbf{a}_y) = (E_z \mathbf{a}_z) \times (+H_3 \mathbf{a}_3) \]

\[ H_3 = \frac{E_z}{P_y} \mathbf{a}_3 \]

Ex:
Find the displacement current at t = 0 through a 10 pF capacitor if the voltage across it is given by

\[ v(t) = 0.18 \sin(120\pi t) \, V \]

\[ C = 10 \, pF \]

\[ I(t)|_{t=0} = ? \quad V \rightarrow pC \]

\[ I = \frac{V}{C} \]

\[ I(0) = \frac{0.18}{10 \times 10^{-12}} \]

\[ I(0) = 1.8 \times 10^{11} \, A \]
\[ \tau = \frac{A}{a} \frac{d \theta}{dt} = e A \frac{e}{\theta} \frac{d \theta}{dt} = \frac{e A \varphi}{e} \frac{d \varphi}{dt} = \frac{e A \varphi}{e} \frac{d \varphi}{dt} \]

\[ c \frac{d}{dt} \left( 0.1 \sin (120 \pi t) \right) = 10 \times 0.1 \times 120 \pi \cos (120 \pi t) \bigg|_{t=0} = 10 \times 12 \pi \]

\[ \varphi = 120 \pi \text{ rad} \]
\[ \varphi = 377 \text{ rad} \]
\[ \varphi = 0.377 \text{ rad} \]

An electromagnetic wave is travelling in a lossless medium is \( \mu = 1 \) & \( \varepsilon = \varepsilon_0 \) & has a power density \( P = \text{W/m}^2 \).

Calculate the max. value of \( E \) & \( H \) phase.

\[ |\vec{E}| = \rho = 4 \text{ W/m}^2 \]
\[ \mu_0 \mu = 1 \text{ j} \]
\[ \mu = \mu_0 \mu = 1 \]
\[ \varepsilon = 4 \text{ j} \]
\[ \varepsilon = \varepsilon_0 \varepsilon = 4 \varepsilon_0 \]

In this:
\[ E_m, H_m \]

\[ P = \frac{1}{2} E_m^2 H_m \]
\[ \eta = \frac{E_m}{H_m} \]
\[ \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{\mu_0}{\sqrt{\varepsilon_0}} \]

\[ P = \frac{1}{2} \frac{E_m^2}{\eta} \]

\[ E_m = \sqrt{2 \pi P} \]
\[ E_m = \sqrt{2 \times 60 \pi \times 4} \]
\[ \eta = \frac{E_m}{H_m} \]
\[ \frac{E_m}{H_m} = \frac{E_m}{60 \pi} = H_m \frac{A}{m} \]
Wave equation

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{1} \]

\[ \nabla \times \vec{H} = \sigma \vec{E} + \frac{\partial \vec{D}}{\partial t} \tag{2} \]

\[ \vec{B}(t) \Rightarrow \frac{\partial \vec{B}}{\partial t} \xrightarrow{\text{eq.}} \vec{E}(t) \rightarrow \vec{H}(t) \]

\[ e = \frac{1}{2} \vec{E}^2, \quad \frac{1}{2} \mu \vec{H}^2 = \omega_m \]

\[ \frac{\partial (\omega_m)}{\partial t} \xrightarrow{\text{eq.}} \frac{\partial (\omega_e)}{\partial t} \]

\[ P_e \xrightarrow{\text{power flows in a direction}} P_m \]

**Conclusion:**

1. Due to time varying ele. & mag. field, the rate of change of ele. energy is transformed to the rate of change of mag. energy.
Due to this rate of change the por propagates on a particular direction which is given by projecting vector \( \mathbf{P} \).

Therefor e.m. waves propagates on the direction given by projecting vector \( \mathbf{P} \).

\[
\text{loge} \left( \frac{V_0}{V_d} \right) = \text{netcess/m}
\]

\[20 \log_{10} \frac{V_0}{V_d} = \text{dB}\]

\[10 \log_{10} \frac{P_0}{P_d} = \text{dB}\]

wave eqn.\[\text{equ.} - 1\]

wave eqn.\[\text{equ.} - 2\]

eliminate \( \mathbf{H} \)

Similarly

\[
\nabla^2 \mathbf{E} = \frac{\mu_0}{\varepsilon_0} \nabla^2 \mathbf{E} + \mu e \frac{\mathbf{E} \cdot \nabla \mathbf{E}}{\nabla^2 } + \frac{\mathbf{E} \cdot \nabla \mathbf{E}}{\nabla^2 } + \frac{\mathbf{E} \cdot \nabla \mathbf{E}}{\nabla^2 }
\]

loss factor

Propagation factor

wave eqn on \( \mathbf{E} \)

in \( \mathbf{H} \)

Comments:

1) Hence has the e.m. waves propagates in general medium it's subjective to attenuation as well as phase change.

Therefore the electric field strength decreases as the wave propagate on a particular direction which is given by projecting vector.
2. The behaviour of elec. & mag. field are exactly same except that the elec. & mag. fields are perpendicular to each other.

Special case:

For sinusoidally varying fields,

\[ \frac{\partial}{\partial t} \Rightarrow j\omega \]

\[ \frac{\partial^2}{\partial t^2} \Rightarrow -\omega^2 \]

\[ \nabla^2 \vec{E} = \mu_0 \cdot j\omega \vec{E} + \mu_e (-\omega^2 \vec{E}) \]

\[ \nabla^2 \vec{E} = \gamma^2 \vec{E} \]

\[ \gamma^2 = \mu_0 \cdot j\omega - \omega^2 \mu_e \]

\[ \gamma = \sqrt{j\omega \mu \left( \sigma + j\omega \mu_e \right)} \]

\[ \gamma = \alpha + j\beta \]

\[ \alpha = 0 \]

\[ \beta = \frac{\omega \sqrt{\mu \mu_e}}{\gamma} = \frac{\omega}{\gamma} \]

\[ \gamma = \frac{1}{\sqrt{\mu_e}} \]
\[ \mathbf{H} = \mathbf{H}_0 \quad \text{for free space} \]

\[ \mathbf{E} = \mathbf{E}_0 \]

\[ V_p = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = C = 3 \times 10^8 \text{ m/sec.} \]

\[ \sqrt{\gamma} = \alpha + j\beta \]

\[ \gamma^2 = -\beta^2 \]

\[ \nabla^2 \mathbf{E} = -\beta^2 \mathbf{E} \]

--- wave eqn for lossless medium \((\sigma = 0)\)

---

**Ex:** An e.m. wave is propagating in a general lossy medium. It has been found that \(\mu, \varepsilon, \sigma\) are constant in the direction of propagation and has only \(x\)-components of electrical field.

Assuming sinusoidal variation, find the value of the electric field components.

**Physical** \(\text{along } +z\)

\[ \mathbf{E} = E_x \hat{\mathbf{a}}_x \]

Medium: \(\mu, \varepsilon, \sigma\)

Sinusoidal variation

To find

\[ E_x \]

\[ \nabla^2 \mathbf{E} = \gamma^2 \mathbf{E} \]

\[ \left( \nabla^2 E_x \right) \hat{\mathbf{a}}_x + \left( \nabla^2 E_y \right) \hat{\mathbf{a}}_y + \left( \nabla^2 E_z \right) \hat{\mathbf{a}}_z = \gamma^2 \left( E_x \hat{\mathbf{a}}_x + E_y \hat{\mathbf{a}}_y + E_z \hat{\mathbf{a}}_z \right) \]
\[ \nabla^2 E_x = \gamma^2 E_x \]

\[ \Rightarrow \frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = \gamma^2 E_x \]

characteristic eqn: \[ m^2 = \gamma^2 \]

\[ m = \pm \gamma \]

\[ E_x = A e^{+\gamma z} + B e^{-\gamma z} \]

\[ E_x = E_0 e^{-\gamma z} \]

**Ans.**

**Depth of penetration:**

\[ E_x = E_0 e^{-\gamma z} \]

\[ = E_0 e^{-(\alpha + j\beta) z} \]

\[ = E_0 e^{-\alpha z} \cdot e^{-j\beta z} \]

\[ |E_x| = E_0 e^{-\alpha z} \]

\[ |E_x| \]

\[ E_0 \]

\[ 0.377 E_0 \]

\[ \Rightarrow z \]

If \( \alpha z = 1 \) for \( z = \delta \)..... depth of penetration

\[ \Rightarrow \delta = \frac{1}{\alpha} \]

\[ \delta = \frac{1}{\alpha} \]

\[ E_x = E_0 e^{-\alpha z} = E_0 e^{-\frac{1}{\alpha z}} \]

\[ \Rightarrow 0.377 E_0 \]
1. As the wave enters in a lossy medium having finite conductivity, the electric field strength decreases exponentially.

2. The depth of penetration or the skin depth represents the distance travelled by electromagnetic waves where the electric field strength decreases to 37% of its initial value.

3. The depth of penetration is inversely equal to the attenuation constant $\alpha$.

4. Higher is the conductivity of the medium, higher is the value of attenuation constant $\alpha$, and therefore lower is the value of depth of penetration; vice versa.

**Case 1:**

For perfect dielectric ($\sigma = 0$),

$$\gamma = \alpha + j\beta = \int j\omega \mu (\sigma + j\omega \epsilon)$$

$$\Rightarrow \alpha = 0$$

$$S = \infty \quad \Rightarrow \frac{1}{\alpha} = \frac{1}{\infty}$$

- perfect dielectric
Case 2: Perfect Conductor
\( (\sigma \to \infty) \)
\[ \Rightarrow \frac{\lambda}{2} \to 0 \]
\[ S = \frac{1}{\lambda} \to 0 \]

\[ \varepsilon = \text{wave} = 0 \]
\[ E = 0 \]
\[ H = 0 \]

**perfect conductor**

\( (\sigma \to \infty) \)

1. For a perfect dielectric \( \sigma = 0, \varepsilon = 0 \), therefore the depth of penetration is infinite.
   In such medium, the wave travels without any attenuation and hence the electric field strength remains constant at all points.

2. For a perfect conductor, the depth of penetration is zero, the wave cannot enter or exit such medium. Therefore, any perfect conductor behaves as a perfect reflector.

3. In this medium, any perfect conductor \( \varepsilon = \mu = 0 \) field and the magnetic field do not exist.

4. Any perfect conductor behaves as an electromagnetic mirror.

5. For a good conductor:
   \[ \frac{\sigma}{\omega \varepsilon} > 1 \]
\[ \gamma = \sqrt{\text{j} \omega \mu_{0} (\sigma + \text{j} \omega \epsilon)} \]

\[ = \sqrt{\text{j} \omega \mu_{0} (1 + \text{j} \omega \epsilon)} \]

\[ = \sqrt{\omega \mu_{0} \left( \cos 45^\circ + \text{j} \sin 45^\circ \right)} \]

\[ = 1/\sqrt{2} = 1/\sqrt{2} \]

\[ \Rightarrow \sqrt{\omega \mu_{0}} (1 + j) = \alpha + j \beta \]

\[ f \uparrow \rightarrow s \downarrow \]

\[ f \downarrow \rightarrow s \uparrow \]

Comments: For a specified material, if the freq of operation is high then the depth of penetration is high. At low freq, since the depth of penetration is high we have to use thin conductor.
Reflection & Refraction of EMW
--- normal incidence

Medium 1
\( \mu_1, \varepsilon_1, \sigma_1, n_1 \)

Medium 2
\( \mu_2, \varepsilon_2, \sigma_2, n_2 \)

\[
\frac{E_i}{H_i} = \left( \frac{E_x}{E_i} \right) = r = \frac{n_2 n_1}{n_2 + n_1}
\]

\[
\frac{H_x}{H_i} = \left( \frac{H_x}{H_i} \right) = r' = \frac{n_1 - n_2}{n_1 + n_2} = -r
\]

\[
T = \frac{2 n_2}{n_2 + n_1}
\]

\[
T' = \frac{2 n_1}{n_1 + n_2}
\]

for general media
\[ \eta = \sqrt{\frac{j \omega \mu}{\sigma + j \omega \epsilon}} = \sqrt{\frac{\mu_0}{\epsilon}} \] 

\[ \eta_1 \propto \frac{1}{\sqrt{\epsilon_1}} \] 

\[ \eta_2 \propto \frac{1}{\sqrt{\epsilon_2}} \] 

\[ \epsilon = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} \] 

\[ T = \frac{2 \sqrt{\epsilon_1}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} \] 

\[ \epsilon' = \frac{\sqrt{\epsilon_2} - \sqrt{\epsilon_1}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} \] 

\[ T' = \frac{2 \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} \] 

\[ \left\{ \begin{array}{l} \epsilon_1 = \epsilon_0 \epsilon_{\text{r}1} \\ \epsilon_2 = \epsilon_0 \epsilon_{\text{r}2} \end{array} \right. \] 

*Is same result*

---

oblique incidence:

incident

\[ \text{reflected} \]

\[ \text{transmitted} \]

\[ \text{incident} \]

\[ \epsilon_{\text{r}1}(\epsilon_1 = 0) \]

\[ \epsilon_{\text{r}2}(\epsilon_2 = 0) \]
\[ \alpha_d = \text{angle of deviation} \]
\[ \alpha_d = \alpha_2 - \alpha_1 \]

\[ \alpha_1' = \alpha_1 \quad \text{--- law of reflection} \]

\[
\frac{\sin \alpha_2}{\sin \alpha_1} = \frac{v_2}{v_1} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}
\]
- Snell’s law

\[ v_1 = \frac{1}{\sqrt{\mu_0 \varepsilon_1}} \]
\[ v_2 = \frac{1}{\sqrt{\mu_0 \varepsilon_2}} \]
\[ \frac{v_2}{v_1} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \]

Case 1: Critical angle

\[ \alpha_c = \alpha_1 \quad \text{when } \alpha_2 = 90^\circ \]

\[ \sin \alpha_c = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \quad ; \quad \varepsilon_2 < \varepsilon_1 \]

\[ \alpha_1 \geq \alpha_c \quad \text{--- total internal reflection (TIR)} \]

\[ \begin{cases} \varepsilon = 1 \\ T = 0 \end{cases} \]

Case 2: Brewster’s angle

\[ \alpha_1 = \alpha_B \]

\[ \tan \alpha_B = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \]

\[ \begin{cases} \varepsilon = 0 \\ T = 1 \end{cases} \]

1. The critical angle represent the angle of incident for which the angle of transmission is 90°.
2. Total internal reflection occurs whenever the angle of incident is more than the critical angle.

This phenomenon is impalpable for the propagation of waves on boundary materials such as propagation through...
The Bragg's angle refers to the angle of incidence for which the wave is not reflected and the emitted wave is transmitted in the second medium. This phenomenon is for the propagation of electromagnetic waves in an unbounded medium, such as the propagation through space or the transmission of waves through a dielectric.

**Boundary Relation for Electric Fields**

\[ \mathbf{E}_{1} \neq \mathbf{D}_{1} \]

\[ \mathbf{E}_{n1} \mathbf{D}_{n1} \]

\[ \mathbf{E}_{n2} \mathbf{D}_{n2} \]

\[ \theta_{1} \]

\[ \theta_{2} \]

\[ \theta_{d} = \theta_{2} - \theta_{1} \]

\[ E_{\text{tangential}} \]

\[ \mathbf{E}_{\text{t1}} \leftrightarrow \mathbf{E}_{\text{t2}} \]

\[ \varepsilon_{s} = C/m^{2} \text{ (surface charge density)} \]
\[ E_{n1} \rightarrow E_{n2} \quad \text{--- normal component relation.} \]
\[ E_1 \rightarrow E_2 \quad \text{--- Oblique incidence of electric fields.} \]

Tangential component relations:

\[ \begin{align*}
\frac{D_{t1}}{\varepsilon_1} &= \frac{D_{t2}}{\varepsilon_2} \\
\hat{a}_n \times (\vec{E}_1 - \vec{E}_2) &= 0
\end{align*} \]

\[ \hat{a}_n \times \vec{E}_1 = E_{1t} \]
\[ \hat{a}_n \times \vec{E}_2 = E_{2t} \]
\[ \hat{a}_n \quad \text{unit normal vector \perp to common boundary.} \]

Case 1: Med. (i) is conduct.
\[ E_{t1} = 0 \]
\[ E_{t1} = E_{t2} = 0 \]

1) The tangential component of electric field intensity \( E \) is continuous across a common boundary separating two different dielectric media.
2) The result is independent of surface charge density present on the common boundary.
3) The tangential components of electric field intensity \( E \) will not exist across a common boundary separating two conducting media when one of them or both media are conducting media.
normal comp. relation

\[\begin{align*}
D_{n1} - D_{n2} &= Es \\
\sin D_{n1} - \sin D_{n2} &= Es \\
\hat{D}_n \cdot (\hat{D}_1 - \hat{D}_2) &= Es
\end{align*}\]

\[\hat{D}_n \cdot \hat{D}_1 = D_{n1}\]
\[\hat{D}_n \cdot \hat{D}_2 = D_{n2}\]

Case 2: \[Es = 0\]
\[D_{n1} = D_{n2}\]

Case 2: Med 2 is cond.
\[D_{n2} = 0\]
\[D_{n1} - D_{n2} = Es\]
\[D_{n1} = Es\]

Case 3: Med 1 is cond.
\[D_{n1} - D_{n2} = Es\]
\[-D_{n2} = Es\]

From Case 2 and 3
\[|D_{n1}| = Es\]
\[|D_{n2}| = Es\]

\[\begin{align*}
\text{Cond} &\quad \text{Cond} \\
Es &\quad c/m^2 \\
|I_m| &= Es
\end{align*}\]
(1) The normal comp. of elec. flux density \( D \) is discontinuous by a factor \( E \) where \( E \) is reconst. surface charge density on the common boundary separating two diff. dielel. media.

(2) for a change free common boundary the normal comp. of elec. flux density \( D \) is always continuous.

(3) Any conducting surface always supports normal comp. of elec. field such that the normal comp. of elec. flux density \( D \) is numerically equal to magnitude of surface charge density present on the conducting surface.

**Oblique Incidence**

\[ \epsilon_3 = 0 \]

\[
\begin{align*}
E_1 &= E_2 \\
D_{n1} &= D_{n2}
\end{align*}
\]

\[
\begin{align*}
E_1 &= E_1 \sin \theta_1 \\
E_2 &= E_2 \sin \theta_2
\end{align*}
\]

\[
\begin{align*}
E_{n1} &= E_1 \cos \theta_1 \\
E_{n2} &= E_2 \cos \theta_2
\end{align*}
\]

\[
\frac{\tan \theta_1}{\tan \theta_2} = \frac{\epsilon_1}{\epsilon_2}
\]
\[
\frac{8 \ln \alpha_1}{8 \ln \alpha_2} = \int \frac{\varepsilon_2}{\varepsilon_1} \, d \alpha
\]

--- Snell's Law

\[\hat{\alpha}_n = + \hat{\alpha}_3\]

\[
\mathbf{E}_1 = \left[\begin{array}{c}
\hat{\alpha}_x = -6 \hat{\alpha}_y - 8 \hat{\alpha}_z
\end{array}\right]
\]

\[\mathbf{E}_2 = \left[\begin{array}{c}
\hat{\varepsilon}_x = 3 \varepsilon_0 \\
- \hat{\varepsilon}_y = \varepsilon_0 \\
- \hat{\varepsilon}_z = \varepsilon_0
\end{array}\right]
\]

\[\mathbf{E}_2 = \hat{\varepsilon}_x = 3 \varepsilon_0
\]

\[\mathbf{E}_2 = \hat{\varepsilon}_y = \varepsilon_0
\]

\[\mathbf{E}_2 = \hat{\varepsilon}_z = \varepsilon_0
\]

To find \[\mathbf{E}_2 = \hat{\varepsilon}_x = 3 \varepsilon_0
\]

\[
\mathbf{E}_2 = \left[\begin{array}{c}
\hat{\varepsilon}_x = 3 \varepsilon_0 \\
- \hat{\varepsilon}_y = \varepsilon_0 \\
- \hat{\varepsilon}_z = \varepsilon_0
\end{array}\right]
\]

\[
\mathbf{E}_2 = \left[\begin{array}{c}
\hat{\varepsilon}_x = 3 \varepsilon_0 \\
\hat{\varepsilon}_y = \varepsilon_0 \\
\hat{\varepsilon}_z = \varepsilon_0
\end{array}\right]
\]

\[\mathbf{E}_2 = \left[\begin{array}{c}
\hat{\varepsilon}_x = 3 \varepsilon_0 \\
\hat{\varepsilon}_y = \varepsilon_0 \\
\hat{\varepsilon}_z = \varepsilon_0
\end{array}\right]
\]

\[\mathbf{E}_2 = \left[\begin{array}{c}
\hat{\varepsilon}_x = 3 \varepsilon_0 \\
\hat{\varepsilon}_y = \varepsilon_0 \\
\hat{\varepsilon}_z = \varepsilon_0
\end{array}\right]
\]

\[\mathbf{E}_2 = \left[\begin{array}{c}
\hat{\varepsilon}_x = 3 \varepsilon_0 \\
\hat{\varepsilon}_y = \varepsilon_0 \\
\hat{\varepsilon}_z = \varepsilon_0
\end{array}\right]
\]

\[\mathbf{E}_2 = \left[\begin{array}{c}
\hat{\varepsilon}_x = 3 \varepsilon_0 \\
\hat{\varepsilon}_y = \varepsilon_0 \\
\hat{\varepsilon}_z = \varepsilon_0
\end{array}\right]
\]

\[\mathbf{E}_2 = \left[\begin{array}{c}
\hat{\varepsilon}_x = 3 \varepsilon_0 \\
\hat{\varepsilon}_y = \varepsilon_0 \\
\hat{\varepsilon}_z = \varepsilon_0
\end{array}\right]
\]

\[\mathbf{E}_2 = \left[\begin{array}{c}
\hat{\varepsilon}_x = 3 \varepsilon_0 \\
\hat{\varepsilon}_y = \varepsilon_0 \\
\hat{\varepsilon}_z = \varepsilon_0
\end{array}\right]
\]

\[\mathbf{E}_2 = \left[\begin{array}{c}
\hat{\varepsilon}_x = 3 \varepsilon_0 \\
\hat{\varepsilon}_y = \varepsilon_0 \\
\hat{\varepsilon}_z = \varepsilon_0
\end{array}\right]
\]

\[\mathbf{E}_2 = \left[\begin{array}{c}
\hat{\varepsilon}_x = 3 \varepsilon_0 \\
\hat{\varepsilon}_y = \varepsilon_0 \\
\hat{\varepsilon}_z = \varepsilon_0
\end{array}\right]
\]

\[\mathbf{E}_2 = \left[\begin{array}{c}
\hat{\varepsilon}_x = 3 \varepsilon_0 \\
\hat{\varepsilon}_y = \varepsilon_0 \\
\hat{\varepsilon}_z = \varepsilon_0
\end{array}\right]
\]

\[\mathbf{E}_2 = \left[\begin{array}{c}
\hat{\varepsilon}_x = 3 \varepsilon_0 \\
\hat{\varepsilon}_y = \varepsilon_0 \\
\hat{\varepsilon}_z = \varepsilon_0
\end{array}\right]
\]

\[\mathbf{E}_2 = \left[\begin{array}{c}
\hat{\varepsilon}_x = 3 \varepsilon_0 \\
\hat{\varepsilon}_y = \varepsilon_0 \\
\hat{\varepsilon}_z = \varepsilon_0
\end{array}\right]
\]

\[\mathbf{E}_2 = \left[\begin{array}{c}
\hat{\varepsilon}_x = 3 \varepsilon_0 \\
\hat{\varepsilon}_y = \varepsilon_0 \\
\hat{v}
A Boundary relations for Mag. Fields

tangential comp. relation

\[ H_{t1} - H_{t2} = J_s \]

\[ \beta H_{t1} - \beta H_{t2} = J_s \]

\[ \frac{\mu_1}{\mu_2} = \frac{H_{t1}}{H_{t2}} \]

\[ \hat{n} \times (H_{t1} - H_{t2}) = J_s \]

\[ \hat{n} \times H_{t1} = H_{t2} \]

\[ \hat{n} \times H_{t2} = H_{t1} \]

\[ H_{t1} = H_{t2} \quad J_s = 0 \]

\[ H_{t1} = J_s \quad \text{if Med.} \odot \text{is cond.} \]

\[ -H_{t2} = J_s \quad \text{if Med.} \odot \text{is cond.} \]

\[ |H_{t1}| = J_s \]

1. The tangential comp. of mag. field intensity \( H \) is a discontinuous by a factor of \( J_s \).

2. \( J_s \) represent the surface current density present on the common boundary separating two different boundary media.

3. On a current carrying face common boundary the tangential comp. of the \( H \) field is always continuous.

Any conducting surface will support only the tangential comp. of \( H \)-field such that the mag.\( \mu_0 \) mag. field intensity \( H \) is numerically equal to the surface current density present on the conducting surface.
Normal comp. relation

\[
B_{n1} = B_{n2}
\]
\[
\mu_1 H_{n1} = \mu_2 H_{n2}
\]
\[
\hat{a}_n \cdot (\overrightarrow{B_1} - \overrightarrow{B_2}) = 0
\]
\[
\hat{a}_n \overrightarrow{B_1} = B_{n1}
\]
\[
\hat{a}_n \overrightarrow{B_2} = B_{n2}
\]

\[
B_{n1} = B_{n2} = 0 \text{ if the medium 1 or med. 2 or both are conductive.}
\]

1. The normal comp. of mag. flux density \( B \) is continuous across a common boundary separating two different mag. media.

2. The result is independent of the surface current density present on the common boundary.

3. The normal comp. of mag. flux density \( B \) will not exist across a common boundary separating two different media, when one of them or both media are conducting media.
Oblique Incidence

\[
\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{u_1}{u_2} = \frac{u_{21}}{u_{42}}
\]

\[\text{Med. (i) } x > 0 \]
\[\mu_1 = 2 \mu_0 \]

\[\vec{B}_1 = 4 \hat{x} + 5 \hat{y} - 6 \hat{z} \]
\[\hat{m} \times \hat{a} \quad J_5 = 0 \]

\[\vec{B}_2 = ? \]

\[\text{Med. (5) } |x| < 0 \]
\[\mu_2 = \mu_0 \]

\[\vec{B}_2 \rightarrow \begin{cases} \text{Normal} \\ \text{tangential} \end{cases} \quad B_{n1} = B_{n2} \rightarrow B_{x2} = B_{x1} = 0 \]

\[H_{d1} = H_{d2} \Rightarrow \frac{\vec{E}_{y2}}{H_2} = \frac{\vec{E}_{y1}}{H_1} \quad ; \quad \vec{E}_{y2} = \frac{u_2}{u_1} \cdot \vec{E}_{y1} \]

\[H_{d2} = H_{d1} \]

\[\Rightarrow \vec{E}_{z2} = \frac{u_2}{u_1} \cdot \vec{E}_{z1} \]

\[\Rightarrow \frac{1}{2} x (-b) = -2 \]

\[\vec{B}_2 = 4 \hat{x} + \frac{35}{2} \hat{y} - 21 \hat{z} \]
Wave polarization

The wave polarization is related to the orientation of the electric field vector associated with the EMW.

If the vertical antenna is installed, the electric field vector is also vertical if the EMW is vertically polarized.

If any antenna is horizontally polarized, the electric field vector is parallel to the surface at the earth. If the EMW is horizontally polarized.

The orientation of the electric field vector at the transmitting & receiving end must be same so that max. induced emf. is obtained at the receiving antenna.

These for the polarization, the transmitting & receiving antenna must be identical.

\[
E_x = E_1 \sin (\omega t - \beta z)
\]

\[
E_y = E_2 \sin (\omega t - \beta z + \phi)
\]

\[\text{Time phase angle } \theta \]

\[\text{Ex & Ey} \]

To bindout:

\[\text{effect of } E_1, E_2, \phi \]

Where \( z = 0 \) plane

\[E_x = E_1 \sin \omega t\]

\[E_y = E_2 \sin (\omega t + \phi)\]
\( \phi = 0^\circ \quad \text{in same phase.} \)
\( \phi = 90^\circ \quad \text{quadrature phase.} \)

**Case 1:** Linear polarization

\[
\begin{align*}
E_1 & \neq E_2 \\
\phi &= 0^\circ \quad \text{or} \quad 180^\circ
\end{align*}
\]

If \( \phi = 0^\circ \):
\[
\begin{align*}
E_x &= E_1 \sin \omega t \\
E_y &= E_2 \sin \omega t
\end{align*}
\]
\[
\frac{E_y}{E_x} = \frac{E_2}{E_1} = m
\]

\[
E_y = mE_x \quad \text{equation of line.}
\]

(a) If \( E_1 = 0 \)
\[
\begin{align*}
E_x &= 0 \\
E_y &= E_2 \sin \omega t
\end{align*}
\]
EMW is linearly polarized along \( y \)-direction.
Wave is vertically polarized.

(b) If \( E_2 = 0 \)
\[
\begin{align*}
E_x &= E_1 \sin \omega t \\
E_y &= 0
\end{align*}
\]
EMW is linearly polarized along the \( x \)-direction.
Wave is horizontally polarized.
Case 2: Circular polarization

\[ E_1 = E_2 = E_0 \]
\[ \chi = \pm 90^\circ \]

\[ E_x = E_0 \sin \omega t \]
\[ E_y = E_0 \sin (\omega t + 90) ; \quad \chi = \pm 90^\circ \]

\[ E_x^2 + E_y^2 = E_0^2 \]
--- eqn of a circle

\[ \chi = +90^\circ \] --- LCP wave
\[ \chi = -90^\circ \] --- RCP wave

--- Elliptical polarization

\[ E_1 \neq E_2 \]
\[ \chi = \pm 90^\circ \]

\[ \chi = +90^\circ \] --- LEP wave
\[ \chi = -90^\circ \] --- REP wave

Main points: 1. When RCP wave incident on a perfect conductor perpendicular, the reflected wave is LCP wave & vice-versa.

RCP (LCP) \[ \rightarrow \] LCP (RCP)
2. Polystyrene (LCP) \[ \rightarrow \text{LEP (REP)} \]

If an RCP wave incidence on polystyrene then:

(a) Transmitted wave is REP wave
(b) Reflected wave is LEP wave & vice-versa

3. If it is a circularly polarized wave for electrically polarized wave incidence at \( \beta \) degree angle on any interface than the reflected wave & transmitted wave are linearly polarized.

\[ \vec{E} = 5 \sin(\omega t - \beta z) \hat{a} \hat{x} + 10 \sin(\omega t - \beta z) \hat{a} \hat{y} \]

\( \beta = 0 \) \( \text{Lineeary polarized} \)

\[ \vec{E} = 5 \sin(\omega t - \beta z) \hat{a} \hat{x} + 5 \sin(\omega t - \beta z - 90) \hat{a} \hat{y} \]

\( \beta = -90 \) \( \text{RCP wave} \)

\[ \vec{E} = 5 \sin(\omega t - \beta z - 30) \hat{a} \hat{x} \\
+10 \sin(\omega t - \beta z + 60) \hat{a} \hat{y} \]

\( \beta = 60 - (-30) = +90 \) \( \text{LEP wave} \)
\[ \frac{d}{dx} \left( 5 \sin(x + \beta) \right) + \frac{d}{dy} \left( 5 \sin(x - \beta) \right) \]
\[ x = 30 - (-30) = 60^\circ \]

unpolarized wave

---

**Example:** An EMW having following \( x \)-compts of elec. field as travelling in a lossless medium with \( n = 1 + \varepsilon_0 = 9 \).

- All the parameters associated with EMW.
- The mag. field intensity associated with EMW.

\[ E = 10 \cdot \cos(5\pi \times 10^3 x - \beta) \hat{\alpha} \]
\[ \sigma = 0 \]
\[ n = 1 \]
\[ \varepsilon_0 = 3 \]

To find:
1. Various parameters
2. \( \frac{E}{H} \)

\[ \tilde{E} = E_0 \hat{\alpha} \]
\[ E_0 = E_0 \cdot \cos(\omega t - \beta) \]
\[ \hat{x} + \hat{z} \text{ direction} \]

\[ E = E_0 \hat{x} \]
\[ E_0 = 10 \text{ V/m} \]

Wave is linearly polarized along \( x \)-direction, direction of propagation along \( +z \) direction.

Surface impedance \( \eta = \sqrt{\frac{E_0}{H}} \)
\[ \eta = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}} = \frac{\mu_0}{\varepsilon_0} = \frac{120\pi}{3} = 40\pi \]

(2) \[ v_p = \frac{1}{\sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}}} = \frac{c}{\sqrt{\varepsilon_0 \varepsilon_r}} = \frac{3 \times 10^8}{10^{12}} = \frac{10^8}{m/\text{sec.}} \]

(3) \[ \omega = 6\pi \times 10^8 = 600\pi \quad \text{Mead/sec} \]

\[ f = \frac{\omega}{2\pi} = 300 \quad \text{MHz} \]

(4) \[ \beta = \frac{\omega}{v_p} = \frac{6\pi \times 10^8}{10^8} = 6\pi \quad \text{rad/m} \]

(5) \[ \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{6\pi} = \frac{1}{3} \quad \text{m} \]

(6) \[ m = \frac{E \cdot H}{\eta} \]

\[ = \frac{1}{2} \eta H_m^2 \]

\[ = \frac{1}{2} \frac{E}{\eta} \cdot \hat{a}_z = \frac{1}{2} \frac{10}{4\pi} \hat{a}_z \quad \text{W/m}^2 \]

(7) \[ n = \frac{E}{H_m} \quad \text{Hm} = \frac{Em}{\eta} = \frac{10}{40\pi} = \frac{1}{4\pi} \quad \text{A/m} \]

(8) \[ E = E \cdot H \]

\[ \hat{a}_z = \hat{a}_x \cdot (+ \hat{a}_y) \]

\[ \hat{H} = H_y \cdot \hat{a}_y \]

\[ H_m \cos \left( \omega t - \beta z \right) \hat{a}_y \]

\[ \frac{1}{4\pi} \cos^8 \left( 6\pi \times 10^8 \cdot t - 6\pi \cdot z \right) \hat{a}_y \]

--- A/m
\[ \mathbf{H} = 10 \sin(5 \pi x + 6 \pi y) \mathbf{a}_z \]

Note: Wave is linearly polarized along \( y \)-direction.

\[ \mathbf{H} = \mathbf{H}_2 \cdot \mathbf{a}_z \]

\[ \mathbf{P} = \mathbf{P}_x \mathbf{a}_x \]

\[ \mathbf{P} = \mathbf{E} \times \mathbf{H} \]

\[ \mathbf{E} = -\mathbf{E}_y \mathbf{a}_y \]

\[ \mathbf{a}_x = \left( \mathbf{a}_y \right) \times (\mathbf{a}_z) \]
Magnetostatics

--- Static Magnetic fields.

\[ \vec{B}, \vec{H} \neq f(t) \]

Ampere's circuit law:

\[ \oint_{c} \vec{H} \cdot \vec{dl} + \frac{1}{c^2} \oint_{c} \vec{B} \cdot \vec{dl} = I \]

(current enclosed)

Ampere's circuit law on a surface current

\[ I = \oint_{c} \vec{J}_s \cdot \vec{dl} = \iint_{S} \vec{J} \cdot d\vec{S} \]

Volume current

\[ \oint_{c} \vec{H} \cdot \vec{dl} = 0 \]
Ampe's circuital law in differential or point form:

\[ \nabla \times \mathbf{H} = J_c + J_d = \sigma \frac{\partial \mathbf{E}}{\partial t} + \frac{\partial \mathbf{D}}{\partial t} \]

\[ \nabla \times \mathbf{H} = \mathbf{J}_c = \sigma \frac{\partial \mathbf{E}}{\partial t} \]

Statements:

1. The circulation of \( \mathbf{H} \), the total current \( \mathbf{I} \) enclosed with the enclosed current \( \mathbf{I} \).
2. If no current is enclosed, the circulation of \( \mathbf{H} \) is along the closed curve is zero all through at each point.
3. The log is applicable irrespective of the shape of the closed curve.
4. The current of static mag field intensity \( \mathbf{H} \) at any point is always equal to conduction current density present at that point.

To find:

\[ \oint_C \mathbf{H} \cdot d\mathbf{A} = ? \]

\[ = 2I + (-2I) = 0 \]

due to flux of current in opp direction on 2 conductors.

due to application of circulation of current.
A long solid cylindrical cylinder, radius \( a \), carries a uniform current \( I \) through out of cross-section of conductor. Find the value of magnetic density \( \vec{B} \) at all points/regions.

\[ \vec{B} \]

1. \( \varepsilon < a \)
   \[ \oint_{c_1} \vec{B} \cdot d\vec{l} = I \]
   
   \[ H_{\phi} \cdot 2\pi\varepsilon = \frac{\varepsilon^2 I}{a^2} \]
   
   \[ H_{\phi} = \frac{I\varepsilon}{2\pi a^2} \]
   
   \[ \vec{B} = \frac{\mu I \varepsilon}{2\pi a^2} \mathbf{\hat{\phi}} \]

2. \( \varepsilon > a \)
   \[ \oint_{c_2} \vec{B} \cdot d\vec{l} = I \]
   
   \[ H_{\phi} \cdot 2\pi\varepsilon = I \]
   
   \[ \vec{B} = \frac{\mu I\varepsilon}{2\pi\varepsilon} \mathbf{\hat{\phi}} \]

\[ \varepsilon < a \quad \varepsilon > a \]
\[ B = \frac{\mu_0 I}{2\pi a} \cdot \hat{\alpha} \]

- **Infopedia**

  - 1. The magnetic field increases linearly on a solid cylindrical conductor whereas it decreases on the hyperbola outside the cylindrical conductor.

  - The magnetic field is always constant on the surface of any conductor.

  - Any cylindrical conductor behaves as an infinite line current since the magnetic field due to both configurations is same.

- **Example**

  A long solid cylindrical conductor of radius \( a \) and conductivity \( \sigma \) carries uniform current \( I \). The magnetic field vector on the surface of cylindrical conductor is given by:

  \[ B = \frac{\mu_0 I}{2\pi a} \cdot \hat{\alpha} \]

  and

  \[ \mathbf{H} = \mathbf{B} \times \hat{\mathbf{r}} \]

  \[ \mathbf{J} = \frac{\sigma}{\mu_0} \mathbf{E} \]
\[ \vec{E} = \frac{J}{\sigma} = \frac{1}{\sigma} \left( \frac{I}{\pi a^2} \right) \hat{a}_3 \]

\[ \vec{p} = \frac{I}{\pi^2 a^2} \int \hat{a}_4 \times \frac{I}{2\pi a} \hat{a}_q \]

\[ = \frac{I^2}{2\pi^2 a^3} \left( \hat{a}_4 \times \hat{a}_q \right) = -\hat{a}_x \]

\[ \vec{p} = \frac{-I^2 \hat{a}_x}{2\pi^2 a^2} \]

**Ex.** A long co-axial TL of radii \( a \) and \( b \) with \( b > a \) carries uniform current \( \pm I \) on the surface of the coaxial cylindrical conductor of the TL.

Calculate magnetic field intensity \( \vec{H} \) at all points.

\[ \text{To find:} \quad \vec{H} \text{ box} \]

1. \( \bar{z} < a \)
2. \( a < \bar{z} < b \)
3. \( \bar{z} > b \)

\[ \oint_{c_1} \vec{H} \cdot d\vec{l} = 0 \]

\[ \Rightarrow \vec{H} = 0 \]

\[ \oint_{c_2} \vec{H} \cdot d\vec{l} = +I + (-I) = 0 \]

\[ \Rightarrow \vec{H} = 0 \]
\[ \int_{c_3} \mathbf{H} \cdot d\mathbf{I} = I \]

\[ \mathbf{H}_\Phi \cdot 2\pi \varepsilon = I \]

\[ \mathbf{H} = \frac{I}{2\pi \varepsilon} \hat{\phi} \]

\[ \frac{\mathbf{H}}{2\pi \varepsilon} = I \hat{\phi} \]

\[ \hat{\phi} \]

\[ \frac{1}{2\pi \varepsilon} \left( \text{hyperbola} \right) \]

\[ I_{1/2\pi \varepsilon} \]

\[ I_{1/2\pi \varepsilon} \]

\[ a < \varepsilon < b \]

\[ \mathbf{H} = 0 \quad \ldots \quad \varepsilon < a \]

\[ = \frac{I}{2\pi \varepsilon} \hat{\phi} \]

\[ = 0 \quad \ldots \quad \varepsilon > b \]
Magnetic energy density

\[ \omega_m = \lim_{\Delta V \to 0} \left( \frac{\Delta \omega_m}{\Delta V} \right) \]

\[ \omega_m = \frac{1}{2} \mu H^2 \]

\[ = \frac{1}{2} \vec{B} \cdot \vec{H} \quad \text{ (J/m}^3\text{)} \]

1. The magnetic energy density represents the magnetic energy stored per unit volume at a point in any electromagnetic region.

2. This depends upon the magnetic field intensity \( H \) and the constant of the medium \( \mu \).

Magnetic energy stored

\[ \omega_m = \frac{1}{2} I I^2 \quad \text{ (J)} \]

Magnetic dipole moment

\[ \vec{m}_m = I A \cdot \hat{n} \quad \text{ (A.m}^2\text{)} \]
Important main points:

1. Moving charge in $\vec{B}$:

\[ \vec{F} = q \vec{v} \times \vec{B} = q u B \sin \theta \, \vec{a}_n \]

\[ |F| = 0 \quad ; \quad \theta = 0^\circ \]

\[ = q u B \quad ; \quad \theta = 90^\circ \]

2. Current carrying cond. on $\vec{B}$:

\[ \vec{F} \times \vec{P} \]
\[ P = \int \mathbf{I} \times \mathbf{B} \, dl \]

\[ \mathbf{F} = \frac{\mathbf{I} \times \mathbf{B}}{l} = N \mathbf{m} \]

- Force per unit length.

3. Carrying loop on \( B \):

- \( F = 0 \)
- \( F_{\text{max}} \)
- \( \theta \)
- \( \mathbf{B} \)
- \( \mathbf{B} \)
- \( \mathbf{I} \mathbf{A} \mathbf{d}_3 \)
- Torque is applicable.

4. Infinite line current:

- \( \beta = \frac{\mu I}{2 \pi z} \)

5. Circular loop:

- \( \beta = \frac{\mu I}{2 \pi} \)
$\mathbf{H}$ or $\mathbf{B}$ due to infinite current sheet:

$$\mathbf{H} = \frac{1}{2} \mathbf{J}_s \hat{a}_n$$

$\hat{a}_n$ → unit normal vector \( \perp \) to current sheet.

At $x = 10$

$$\hat{a}_n = \hat{a}_x$$

$$\mathbf{H} = \frac{1}{2} \mathbf{J}_s \times \hat{a}_n = \frac{1}{2} \mathbf{J}_s \hat{a}_n \times (\hat{a}_x)$$

$$= \frac{1}{2} \mathbf{J}_s \hat{a}_y$$

At $x = 2$

$$\hat{a}_n = -\hat{a}_x$$

$$\mathbf{H} = \frac{1}{2} \mathbf{J}_s \times \hat{a}_n = \frac{1}{2} \mathbf{J}_s \hat{a}_y \times (-\hat{a}_x)$$

$$= -\frac{1}{2} \mathbf{J}_s \hat{a}_y$$

7) Coaxial T.L.:
\[
\frac{1}{c} \left[ \frac{1}{2\pi} \int \frac{\mu}{\ln \left( \frac{b}{a} \right)} \right] - \frac{1}{\ln \left( \frac{b}{a} \right)} - \frac{2\pi \varepsilon}{\ln \left( \frac{b}{a} \right)} - f \frac{\mu}{\varepsilon}
\]

\[Z_0 = \sqrt{\frac{1}{c^2} = \frac{1}{2\pi} \sqrt{\frac{\mu}{\varepsilon} \ln \left( \frac{b}{a} \right)}} \frac{\varepsilon}{\mu} \ln \left( \frac{b}{a} \right) \]

For free space:

\[\varepsilon = \frac{\varepsilon_0}{\mu} \]

\[Z_0 = \frac{60 \ln \left( \frac{\varepsilon}{\mu} \right)}{\Omega} \]

\[v_p = \frac{1}{\sqrt{\frac{1}{2} \varepsilon}} \]

\[v_p = \frac{1}{\sqrt{\mu}} \]

For free space:

\[v_p = \frac{1}{\sqrt{\varepsilon_0}} = c \]

\[\mu = \mu_0 \]

\[\varepsilon = \varepsilon_0 \]
Vector Mag. Potential ($\vec{A}$)

1. $\nabla \cdot \vec{B} = 0$  \[\nabla \cdot (\nabla \times \vec{A}) = 0 \quad \text{always valid.} \]

2. $\vec{B} = \nabla \times \vec{A}$  \[\vec{B} \perp \vec{A} \quad \text{vector identity.} \]

Mathematical definition of $\vec{A}$:

$$
\vec{A} = \frac{\mu}{4\pi} \int_c \frac{\vec{I}}{r^2} \, dl
$$

$\Rightarrow \vec{A} \parallel \vec{I}$

Direction-wise:

- $\vec{A} \parallel \vec{I}$
- $\vec{B} \parallel \vec{I}$
- $\vec{B} \perp \vec{A}$

$\vec{B} = \nabla \times \vec{A}$

1. There is no physical significance of vector mag. potential $\vec{A}$ since the mag. charges on the insulated sheet do not exist in nature.

2. Calculation of vector $\vec{A}$ simply gives
an intermediate step.
  to calculate the value of mag. 
flux density $B$, using the relation

$$\vec{B} = \nabla \times \vec{A}.$$  (95)

\[ \vec{A} = 2x^2y \hat{a}_z \]
To bind
\[
\vec{B} = \nabla \times \vec{A}
\]
\[
\vec{B} = \begin{vmatrix}
\hat{a}_x & \hat{a}_y & \hat{a}_z \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
A_x & A_y & A_z \\
\end{vmatrix} = \hat{a}_x \frac{\partial}{\partial y} \left( 2x^2y \right) + \hat{a}_y \left[ -\frac{2}{\partial x} \left( 2x^2y \right) \right] + \hat{a}_z \left( 0 \right)
\]
\[ = 2x^2 \hat{a}_x - 4xy \hat{a}_y \]
\[
\nabla \times \vec{B} = 0 \quad \text{irrotational vector.}
\]
\[ \neq 0 \quad \text{not an} \]

units of $\vec{A}$

\[ \vec{B} = \nabla \times \vec{A} \quad \Rightarrow \quad \frac{Wb}{m} \frac{1}{m} \]
**Electrostatics**

**Static Electric Fields**

\[ \vec{E}; \vec{D} \neq f(t) \]

**Coulomb’s Law:**

\[ \vec{E} = \frac{q}{4\pi\varepsilon_0 r^2} \]

\[ \vec{E} = \frac{q}{4\pi\varepsilon_0} \hat{a}_r \]

\[ a_x = \frac{x}{r^2} \]

\[ a_y = \frac{y}{r^2} \]

\[ a_z = \frac{z}{r^2} \]

\[ \vec{E} = \frac{q}{4\pi\varepsilon_0} \frac{\hat{a}_x (x-x') + \hat{a}_y (y-y') + \hat{a}_z (z-z')}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}} \]

A point charge of +3 nC and coloumb is placed at a point \((2, 1, -2)\) in a medium whose dielectric constant is 3.

To find the electric field at point \((1, 3, -1)\):

\[ \vec{r} = \text{radius vector} \]

\[ (2, 1, -2) \]

\[ (1, 3, -1) \]

\[ \phi = +3 \text{nC} \]

\[ \varepsilon_0 = \varepsilon \]

\[ \varepsilon = 3 \]

To find the electric field at point \((1, 3, -1)\):}

\[ \vec{E} = \frac{q}{4\pi\varepsilon_0 \varepsilon} \]

\[ \vec{E} = \left( \frac{3}{4\pi\varepsilon_0 \varepsilon} \right) \left( \frac{1 - 2}{1} \hat{a}_x + \frac{3 - 1}{1} \hat{a}_y + \frac{-2 + 1}{1} \hat{a}_z \right) \]

\[ \vec{E} = \left( \frac{3}{4\pi\varepsilon_0 \varepsilon} \right) \left( \hat{a}_x - \hat{a}_z \right) \]
\[ \dot{\rho} = -\dot{\alpha} x + 2 \dot{\alpha} y + \dot{\alpha} z \]

\[ |\dot{\rho}| = |1 + 4 + 1| = \sqrt{6} \]

\[ E = \frac{3 \times 10^3}{4 \pi \times \frac{1}{3} 6 \pi} \times 10^{-8} \times 3 (\sqrt{6})^3 \]

\[ E = \frac{9^3}{3 \times 6 \sqrt{6}} \]

\[ E = E \hat{a}_1 \]

\[ \hat{a}_1 = \frac{E}{E} = \frac{K}{3/2} (-\dot{\alpha} x + 2 \dot{\alpha} y + \dot{\alpha} z) \]

\[ \hat{a}_1 = \frac{1}{\sqrt{6}} (-\dot{\alpha} x + 2 \dot{\alpha} y + \dot{\alpha} z) \]

\[ \alpha = \cos^{-1} \left( \frac{E}{E} \right) = \cos^{-1} \left( \frac{1}{\sqrt{6}} \right) \quad \text{w.r.t. x-axis} \]

\[ \beta = \cos^{-1} \left( \frac{E_y}{E} \right) = \cos^{-1} \left( \frac{9}{\sqrt{6}} \right) \quad \text{w.r.t. y-axis} \]

\[ \gamma = \cos^{-1} \left( \frac{E_z}{E} \right) = \cos^{-1} \left( \frac{1}{\sqrt{6}} \right) \quad \text{w.r.t. z-axis} \]
\[ \Phi = \oint_S \mathbf{D} \cdot d\mathbf{s} = \Phi \]  
--- in integral form

\[ \text{net elec. flux} \]

\[ \Phi = \Phi \quad \text{--- point charge} \]
\[ = \int_C \mathbf{E} \cdot d\mathbf{c} \quad \text{--- line charge} \]
\[ = \iint_S \mathbf{E} \cdot d\mathbf{s} \quad \text{--- surface charge} \]
\[ = \iiint_V \mathbf{E} \cdot d\mathbf{V} \quad \text{--- volume charge} \]

\[ \nabla \cdot \mathbf{D} = \mathbf{0} \quad \text{--- differential form of point form} \]

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]

\[ \nabla \times \mathbf{E} = \mathbf{0} \quad \text{--- isototnal static elec field} \]

\[ \mathbf{J}_e \cdot d\mathbf{A} = 0 \]

**points:**

1. The net elec. flux passing through any closed surface area is always equal to total charge enclosed by the closed surface with area \( S \).

The eqn. is valid for any type of the closed surface with area \( S \).

The divergence of elec. flux density \( \mathbf{D} \)
at any point if the electro-mag. region is
always equal to the volume charge density at
that point.

4) The curl of static elec. field intensity \( \mathbf{E} \) at
any point is always zero & therefore for such
field is always rotational.

therefore the circulation of the static
elec. field along any closed curve \( \mathbf{C} \) is
always equal to zero.

\[
\varepsilon_0 = \text{C/m}^2
\]

To find \( \mathbf{E} \), \( \mathbf{D} \)
box \( a < z < b \)

\[
\begin{align*}
\mathbf{E} &= \frac{\varepsilon_0 \cdot a}{\varepsilon} \\
\mathbf{D} &= \varepsilon_0 \mathbf{E}
\end{align*}
\]

\[ \iint_S \mathbf{D} \cdot d\mathbf{S} = \Phi = \iiint_S \mathbf{E} \cdot d\mathbf{S} \]

\[ E_z = \frac{\varepsilon_0 \cdot a}{\varepsilon} \]

ex.:

A charge +\( \Phi \) is distributed throughout the
volume of region of radius \( \text{a} \). Find the value of electro-
static intensity \( \mathbf{E} \) at all point.
To bind

1. \( z < a \)
2. \( z > a \)
3. \( z = a \)

\[ \Phi = \frac{q e^2}{4 \pi \varepsilon_0 a^3} \]

\[ \vec{E} = \frac{q e^2}{4 \pi \varepsilon_0 a^3} \frac{\hat{z}}{a} \]

\[ E_x = \frac{q e^2}{4 \pi \varepsilon_0 a^3} \]

\[ E_z = \frac{q e^2}{4 \pi \varepsilon_0 a^3} \]

\[ E_x < \frac{q}{4 \pi \varepsilon_0 a^2} \]

\[ E_z > \frac{q}{4 \pi \varepsilon_0 a^2} \]

\[ E_z = \frac{q}{4 \pi \varepsilon_0 a^2} \hat{z} \]
Two spherical cells of radii 'a' and 'b'. With b greater than a and equal & opposite charges ±q on their surfaces.

bind elec field intensity \( E \) at all points.

To bind:

\( E = \frac{Q}{4\pi \varepsilon_0 a^2} \)

1. \( a < b \)
2. \( a < q < b \)
3. \( q > b \)

\[
\begin{cases}
  a < q < b ; & E = \frac{Q}{2\pi \varepsilon_0 b}\ \\
  a > q ; & E = 0
\end{cases}
\]
Electric energy density

\[ W_e = \lim_{\Delta V \to 0} \frac{\Delta W_e}{\Delta V} \]

\[ W_e = \frac{1}{2} \varepsilon E^2 \]

\[ = \frac{1}{2} \varepsilon \nabla \cdot E^2 = -\frac{1}{2} \Phi^2 \]

Electric energy stored

\[ W_e = \frac{1}{2} CV^2 \]

\[ = \frac{1}{2} \varepsilon V \]

\[ = \frac{1}{2} \varepsilon \]

Since \( \varepsilon = C \cdot V \)

**Electric energy stored in a system of 2 charges**

\[ W_{e1} = \frac{1}{2} q_1 V_2 = \frac{1}{2} q_1 \frac{q_2}{4 \pi \varepsilon_0} \]

\[ W_{e2} = \frac{1}{2} q_2 V_1 \]

\[ W_{e} = W_{e1} + W_{e2} \]

\[ W_e = \frac{q_1 q_2}{4 \pi \varepsilon_0} \]
\[ \nabla \cdot \vec{D} = \rho \quad \text{--- Gauss' Law} \]
\[ \Rightarrow \epsilon \nabla \cdot \vec{E} = \rho \]
\[ \Rightarrow \vec{E} = - \nabla \phi \]
\[ \Rightarrow \nabla \cdot \nabla \phi = \frac{\rho}{\epsilon} \quad \text{--- Poisson's Equation} \]

\[ \nabla^2 \phi = 0 \quad \text{--- Laplace Equation} \]

\[ \text{Div} \vec{V} = 0 \text{ } \text{Div} \vec{E} = - \nabla \phi \]

**Points:**
1. The Poisson equation represents 2nd order 3-dimensional non-homogeneous differential equation.
2. The Laplace equation represents 3-dimensional 2nd order homogeneous differential equation.
3. Using these equations, the potential \( V \) and electric field \( \vec{E} \) can be found due to a specified volume charge distribution or a charge-free region.
To find:

Relation b/w

\[ V_1, V_2, V_3 \]

\[ \nabla^2 V = 0 \]

\[ V = f(x, y, z) \]

In general

\[ V = f(x) \]

only

\[ \nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \]

\[ \frac{\partial V}{\partial x} = A \; ; \quad V = Ax + B \]

Boundary conditions:

1. \( x = 0 \); \( V = V_1 \)
2. \( x = d \); \( V = V_2 \)
3. \( x = 3d \); \( V = V_3 \)

\[ 1 \quad V_1 = A0 + B \Rightarrow B = V_1 \]

\[ 2 \quad V_2 = Ad + B = Ad + V_1 \Rightarrow A = \frac{V_2 - V_1}{d} \]

\[ V = Ax + B \]
\[ V = \frac{V_2 - V_1 \cdot \alpha + V_1}{\alpha} \]

\[ V_3 = \frac{V_2 - V_1}{\alpha} \cdot 3\alpha + V_1 \]

\[ V_3 = 3V_2 - 2V_1 \]

**Example:**

\[ V = 3x^2 \cdot y^4 \]

To bind:

\[ \epsilon \left( 1, 1, 1 \right) \]

\[ \nabla^2 V = -\frac{\epsilon}{\epsilon} \]

\[ \Rightarrow \epsilon = -\frac{\epsilon}{\nabla^2 V} \]

\[ \epsilon = -\epsilon \left[ \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right] \]

\[ \frac{\partial V}{\partial x} = 12x^2y^4 \quad ; \quad \frac{\partial^2 V}{\partial y^2} = 36x^2y^2 \]

\[ \epsilon = -\epsilon \left[ 36x^2y^4 + 36x^2y^2 \right] \]

\[ \epsilon \left( 1, 1, 1 \right) = -\epsilon \left[ 36 \right] \quad \text{in } \text{m}^3 \]

\[ \overrightarrow{E} = 2x^2y^2 \overrightarrow{0z} \Rightarrow E_x \overrightarrow{0z} \]

To bind:

\[ \epsilon \left( 1, 1, 1 \right) \]

\[ \phi = 0 \quad \epsilon_x = 0 \quad \epsilon_y = 0 \quad \epsilon_z = \nabla \cdot \overrightarrow{D} \]

\[ \overrightarrow{E} = \text{Gauget} \]

\[ \overrightarrow{D} = \epsilon \overrightarrow{E} \]

\[ E = \nabla \cdot \overrightarrow{D} \]
\[
\vec{D} = \epsilon_0 \varepsilon r^2 \hat{a}_z = \varepsilon \varepsilon_0 \hat{a}_z
\]

\[
\vec{P} = \nabla \cdot \vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \frac{\partial}{\partial z} \left[ \varepsilon \varepsilon_0 \frac{\partial}{\partial z} \right] = 0
\]

\[
\nabla \cdot \vec{D} = \vec{P} = 0
\]

--- Change Free Region
--- \( \vec{D} \) is solenoidal

\[
V = 4\pi^2 y
\]

\( \vec{E} = -\nabla V = -\text{Grad} \, V \)

\[
= - \left[ \frac{\partial V}{\partial x} \hat{a}_x + \frac{\partial V}{\partial y} \hat{a}_y + \frac{\partial V}{\partial z} \hat{a}_z \right]
\]

\[
= - \left[ 8\pi y \hat{a}_x + 4\pi^2 \hat{a}_y \right]
\]
Method of Image

Another method to find

\[ V \cdot E = q \]

\[ V = \frac{E}{\mu} \]

\[ \text{Point Charge} \]

\[ \text{infinite Cond.} \]

\[ \text{grounded Cond.} \]

Summary

\[ V \cdot E = -q \]

\[ \text{Point Charge} \]

\[ z \cdot V \cdot E = -V \]

Theory of Images

Points:

1. The method of images is applicable to the e.m. problems where any point charge is placed at some height above an infinite cond. or a grounded cond.

2. Any perfect conductor behaves as a perfect reflector and there for acts as electromagnetic mirror.

The entire theory of optics is applicable.

The theory of images is applicable only
to the e.m. problems. For magnetic field theory is not applicable. Since the magnetic changes on the isolated form do not exist in nature.

Ex: A point charge $+Q$ is placed at a height $H$ above a grounded conductor. Calculate the surface charge density induced on the conducting sheet.

\[ E = \frac{Q}{4\pi \varepsilon_0 r^2} \]

\[ D = \varepsilon E \]

\[ D_n = \frac{1}{\varepsilon} E_s \]

\[ E = E_1 + E_2 \]

\[ E_1 = E_0 = \frac{Q}{4\pi \varepsilon_0 r^2} \]

\[ E_2 = \frac{Q}{4\pi \varepsilon_0 (r^2 + h^2)} \]

\[ E_n = |E_1| \cos \alpha + |E_2| \cos \alpha \]

\[ = 2 E_0 \cos \alpha \]
\[ E_n = \frac{\varphi h}{2\pi \epsilon \left(x^2 + h^2\right)^{3/2}} \]

\[ E_n = -e E_n = \left| \mathcal{E}_s \right| = \frac{\varphi h}{2\pi \left(x^2 + h^2\right)^{3/2}} \]

\[ \mathcal{E}_s (\text{ind}) = \frac{-\varphi h}{2\pi \left(x^2 + h^2\right)^{3/2}} \]

---

**Example:**

A point charge \( +\varphi \) is placed in front of a grounded spherical conductor of radius \( R \) as shown.

Calculate the magnitude and the location of the image charge.

**Images on Spheres**

![Diagram of a grounded spherical conductor with an image charge](image)
\[ q^1 = \text{Magnitude} \] ab image
\[ a = \text{location} \] change

\[ V = \frac{q}{4 \pi e_0} \]

\[ V_{p1} = \frac{1}{4 \pi e_0} \left( \frac{q}{d-R} + \frac{q^1}{R-a} \right) = 0 \]

\[ V_{p2} = \frac{1}{4 \pi e_0} \left( \frac{q}{d+R} + \frac{q^1}{R+a} \right) = 0 \]

\[ \frac{q}{q^1} = -\frac{d-R}{R-a} \]

\[ \frac{q}{q^1} = -\frac{d+R}{R+a} \]

\[ \frac{d-R}{R-a} = \frac{d+R}{R+a} \]

\[ a = \frac{R^2}{d} \]

\[ \frac{q}{q^1} = -\frac{d-R}{R-a} \]

\[ q^1 = -\frac{qR}{d} \]
### Quadrupole

<table>
<thead>
<tr>
<th>Type of Configuration</th>
<th>( \nabla )</th>
<th>( \vec{E} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monopole</td>
<td>( 1/2 )</td>
<td>( 1/e^2 )</td>
</tr>
<tr>
<td>Dipole</td>
<td>( 1/\pi^2 )</td>
<td>( 1/\pi^3 )</td>
</tr>
<tr>
<td>Quadrapole</td>
<td>( 1/\gamma^3 ), ( 1/\gamma^4 )</td>
<td></td>
</tr>
<tr>
<td>Octopole</td>
<td>( 1/\psi^4 ), ( 1/\psi^5 )</td>
<td></td>
</tr>
</tbody>
</table>
Main points:

1. Electric dipole

\[ \begin{array}{c}
+q & -q \\
\text{at} & 0 \\
\text{d} & \end{array} \]

\[ \vec{E} = \frac{\vec{q} \cdot \vec{r}}{\epsilon_0 r^2} \]

Dielectric moment

\[ \vec{m} = qd \cdot \hat{a}_1 \]

2. \( \vec{E} \) due to infinite line charge

\[ \vec{E} = \frac{q / \mu_0}{2\pi \epsilon_0} \]

3. Circular loop

\[ \begin{array}{c}
+ & \\
\text{at} & 0 \\
\text{c/m} & \end{array} \]

\[ \begin{cases}
\nabla V = \text{const}
\\
\vec{E} = 0
\end{cases} \]
4) Infinite charge sheet:

\[ E = \frac{\varepsilon_0 \rho}{2 \varepsilon \varepsilon_0} \]

\[ E = \frac{\varepsilon_0}{2 \varepsilon} \frac{\rho}{\varepsilon_0} \]

\[ E = \frac{\varepsilon_0 \rho}{2 \varepsilon \varepsilon_0} \]

\[ E = \frac{\rho}{2 \varepsilon} \]

5) Concentrating spherical sheets:

\[ E = \frac{4 \pi \varepsilon \varepsilon_0}{b-a} \]

6) Cylindrical transmission line:

\[ E = \frac{2 \pi \varepsilon}{\ln(b/a)} \]

\[ E = \frac{\varepsilon_0}{2 \varepsilon} \frac{\rho}{\varepsilon_0} \]
Solution Chapter 3 (T.T.)

1. \[ V_{m} = 15V \]
   \[ Z_{L} = Z_{o} \]
   \[ V = V_{m} \]
   \[ Z_{o} = 50 \Omega \]
   \[ Z_{L} = Z_{o} \frac{1 + \Gamma}{1 - \Gamma} = 36.6 \Omega \]
   \[ \Gamma = \frac{1}{1 + \frac{Z_{L}}{Z_{o}}} \]

2. \[ V_{L} = \frac{V}{1 + \Gamma} = 1.268 \]
   \[ V_{L} = 15 \times 1.268 = 19V \]
   \[ Z_{o} V_{p} = \frac{1}{C} \]
   \[ Z_{o} = \sqrt{\frac{1}{C \cdot V}} \]

3. \[ f = 10 \text{ GHz} \]
   \[ d = 3 \times 10^{-3} \text{m} \]
   \[ \phi = 90^\circ = \frac{\pi}{2} \]

4. \[ \phi = \beta d \]
   \[ \theta = \frac{\omega d}{V_{p}} \]
   \[ Z_{m} = Z_{a1} || Z_{a2} \]
   \[ = Z_{1} || Z_{1} \]
   \[ = 25 \Omega \]
\[ \begin{align*}
20 & \quad \ell = \frac{27.5}{12.5} = 2.2 \text{ cm} \\
21 & \quad f = \frac{c}{a} = \frac{3 \times 10^7}{30} = 1 \text{ kHz}
\end{align*} \]

\[ \begin{align*}
22 & \quad S = 3 \\
23 & \quad f = \frac{V}{V_i} = \frac{E_i}{E_i} = \frac{S-1}{S+1} = 1/2 \\
24 & \quad \varepsilon_\ell = 1/4 \geq 25\%
\end{align*} \]

reflection coeff of power

\[ I_L \rightarrow I_S \]

\[ V_L = \cos \phi \beta \cdot V_S = j \frac{Z_0 \sin \beta L \cdot I_S}{300} \]

\[ |V_L| = \left| -j \frac{300}{3} \right| = 60 \text{ V} \]

26

\[ Z_0 = \sqrt{Z_{oc} \cdot Z_{sc}} \]

\[ Z_{sc} = \frac{-Z_0^2}{Z_{oc}} = \frac{100 + j150}{\text{inductive}} \]

27
\[ S = \frac{V_{\text{max}}}{V_{\text{min}}} = \frac{4}{1} = 4 \]

\[ Z_{\text{min}} = \frac{Z_o}{S}, \quad Z_{\text{max}} = Z_o \cdot S \]

\[ S = \frac{1 + e^{-1 + \sqrt{3}}}{1 - e^{-1 - \sqrt{3}}} = \frac{4}{\sqrt{3}} \]

\[ T = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{s - 10}{s + 10} = \frac{1}{s} \]

\[ a < \xi < b \]

\[ Z_{\text{in}} = Z_{1/2} || Z_{1/6} \]

\[ Z_L = 100 \Omega \]

\[ Z_{\text{in}} = 100 || 50 \]

\[ Z_{\text{in}} = \frac{1 + \frac{j}{10}}{10} \]

\[ H = \sin(5 \times 10^4 t + 0.00012 \omega + \beta) \]

\[ \theta_p = \frac{-\omega}{\beta} \]
\[ V_0 = V + V_0 = 10 \]

\[ V_0 = 10 \]

\[ i_{ss} = \frac{1}{2} \left( I_{max} + I_{min} \right) \]

\[ I_{max} = \frac{V_{max}}{R_L} = \frac{V^+ + V^-}{R_L} = \frac{100}{100} = 1 \text{A} \]

\[ I_{min} = \frac{V_{min}}{R_L} = \frac{V^- - V^+}{R_L} = \frac{20}{100} = 0.2 \text{A} \]

\[ i_{ss} = \frac{1}{2} \left( 0.4 + 0.2 \right) = 0.3 \text{A} \]

\[ Z_0 = 50 \Omega \]

\[ \begin{bmatrix} g_{11} \\ g_{12} \\ g_{21} \\ g_{22} \end{bmatrix} \]

\[ g_{11} = \text{total power sent from port 11' to port 22'} \]

\[ g_{12} = \text{the total power sent back to port 11' from port 22'} \]

\[ \begin{bmatrix} g \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \]

\[ \eta = \frac{\text{loss}}{\text{power}} \approx 1 \]

\[ \eta = \sqrt{\frac{1}{2} \frac{(1 + x)^2}{2x}} \]

\[ \eta = \sqrt{\frac{1}{2} \frac{1}{1 + x}} \]

\[ \eta = \sqrt{\frac{x}{x + 1}} \]
\[ E_i = \cos \left( \omega t - \beta z \right) \hat{y} \]

\[ E_i = e^{j \omega t} \hat{y} \]

\[ \text{Propagation in a direct line} \]

\[ E^0 \text{ has only } \hat{y} \text{ component.} \]

\[ w = 3 \times 10^9 \pi \text{ - rad/sec.} \]

\[ \beta = 10\pi \text{ - rad/m} \]

\[ v_p = \frac{w}{\beta} = 3 \times 10^8 \text{ m/sec.} \]

\[ c = \frac{v_p}{\beta} \]

\[ \text{Med 0 is free space} \]

\[ E_0 = E_0 \left( \frac{\partial}{\partial t} \right) \text{ Prob. + 2y} \]

40

\[ \text{Skin depth } s = \frac{1}{j\omega \mu_0 \sigma} \]

\[ \mu_0 = 4\pi \times 10^{-7} \]

42

\[ Z_{in} = 72 \Omega \]

\[ Z_0 \]

\[ Z_L = 50 \Omega \]

\[ Z_0 \]

\[ Z_L = \frac{Z_{in} \cdot Z_L}{\sqrt{Z_{in} \cdot Z_L}} = \sqrt{72 \times 50} = 60 \Omega \]

\[ Z_0 = \ln \left( \frac{b}{a} \right) = 60 \]

\[ Z_0 = \ln \left( \frac{b}{a} \right) \Rightarrow \frac{b}{a} = 10^1 \frac{b}{a} \approx 2.7 \]

\[ \frac{2b}{2a} \approx 2.7 ; \quad 2b \approx (2.7) 29 \]

\[ 2b - 24 \text{ mm} \]

\[ 2b - 24 \text{ mm} \]
\[
\gamma = \frac{0.0005 + j \frac{\pi}{10}}{\alpha + j \beta} \]

\[
\alpha L = (0.0005 \times 50) \cdot \text{measured}
\]

\[
\times 8.686
\]

\[
\downarrow \quad \text{dB}
\]

\[
L = 2 \Rightarrow L = \frac{220}{2}
\]

\[
R_0 = \frac{Z_L}{2}
\]

\[
Z_{L1} = \frac{Z_{L2}}{2} \Rightarrow 2R_0
\]

\[
Z_{L2} = \frac{Z_{L3}}{2} \Rightarrow \frac{2R_0}{R_0/2}
\]

\[
Z_{in} \text{ at } 218 = Z_0 = R_0
\]

\[
\text{perfectly matched} = 2R_0 || 2R_0 = R_0
\]

\[
R, G_1 = \text{small}
\]

\[
\gamma = \alpha + j \beta = \sqrt{(R + j \omega L)(G_1 + j \omega C)}
\]

\[
R \ll j \omega L, \quad G_1 \ll j \omega C
\]

\[
J \omega L C \left[ 1 + \frac{R}{j \omega L} + \frac{G_1}{j \omega C} \right] - \frac{R_0}{j \omega L C} \right]^{1/2}
\]

\[
(1 + \gamma)^{1/2}
\]

\[
\gamma = J \omega L C \left[ 1 + \frac{1}{2} \left( \frac{R}{j \omega L} + \frac{G_1}{j \omega C} \right) \right]
\]
Real part
\[ \alpha = \omega \sqrt{\frac{R}{2\omega L} + \frac{g_1}{2\omega C}} \]
\[ \alpha = \frac{1}{2} \left( \frac{R}{Z_0} + \frac{g_1}{Z_0} \right) \]
\[ \alpha = \frac{1}{2} \left( \frac{R}{Z_0} + g_1 Z_0 \right) \]

46
\[ l = 0.5 \text{m} \]
\[ L = \text{---} \]
\[ C = \text{---} \]
\[ f = 25 \text{MHz} \]
\[ B1 = \frac{\omega}{\nu p} \]
\[ l = \frac{2\pi f l}{\nu p} = \frac{2\pi f l}{1/\sqrt{LC}} \]

51
\[ Z_0 = 50 \Omega \]
\[ Z = 0.5 - j0.3 = \frac{Z}{Z_0} \]
\[ Z = (0.5 - j0.3) Z_0 \]
\[ \boxed{Z = Z_0} \]

In Smith chart
draw all the impedance
be taken in normalized form
2. \( d = \frac{a}{4} = \frac{1}{4} \frac{v_p}{f} = \frac{1}{4} \frac{c}{f \varepsilon} \) \( \Rightarrow 0.28 \)

3. \( \eta = \sqrt{\frac{\mu}{\varepsilon}} \quad v_p = \frac{1}{\sqrt{\mu \varepsilon}} \)

6. Dielectric Space - Perfect Dielect.

\[ \sigma = 0 \]

\[ \varepsilon = \varepsilon_0 \frac{1}{1 + \varepsilon} \]

\[ \varepsilon = \varepsilon_0 \]

\[ \mathbf{J}_e = \sigma \mathbf{E} = 0, \quad \mathbf{P}_c = 0 \]

\[ \mathbf{P} = 0 \]

\[ v_p = c = 3 \times 10^8 \text{ m/sec.} \]

\[ \eta = \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120 \pi \Omega \]

11. \[ H = 0.1 \sin(10^8 \pi t + \beta y) \quad a_z \]

\[ \mathbf{P} = \frac{1}{2} \eta_0 \mathbf{H}^2 \]

\[ \frac{1}{2} \times 120 \pi \times 0.1 \]

15. \( \sigma = 5 \text{ mho/m} \)

\( \varepsilon = 80 \)

\( f = 25 \text{ kHz} \)

\[ \text{low freq.} \quad \text{low freq.} \]

\[ E_z = E_0 e^{-kz} \quad e^{-kz} = \frac{E_z}{E_0} = 0.1 \]

\[ e^{+kz} = 10 \]

\[ R = \frac{1}{\alpha} \text{ ln}(10) \]

\[ \frac{\sigma}{\omega \varepsilon} \Rightarrow \text{Good Conductor} \]

\[ \chi = \sqrt{\frac{\omega \mu_0}{\alpha}} - \sqrt{\eta \mu_0} = \mu_0 \]
(9) \[ E_z = E_0 e^{-j(\gamma + \beta)z} \]

(10) \[ E = \frac{\omega}{j} \psi \frac{d}{dy} (18y + K \cdot z) \]

\[ K = \frac{\omega}{\mu} = \frac{10^7}{3 \times 10^8} = \frac{1}{30} \]

\[ a = \frac{2\pi}{\beta} = 2\pi \times 30 = 60\pi \approx 188.5 \text{ m} \]

(11) \[ \mu = \varepsilon_1 = \eta_0 = 8.85 \times 10^{-12} \text{ m}^{-1} \]

\[ \eta_2 = \frac{\mu}{\varepsilon} = \frac{1}{\varepsilon} \Omega \]

\[ \eta = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{1 - 8.85 \times 10^{-12}}{1 + 8.85 \times 10^{-12}} \approx -1 \]

(22) \[
\begin{cases}
\hat{B}(t) \\
\text{Ind}
\end{cases}
\]

\[ \hat{B}(t) \rightarrow \psi m(t) \rightarrow \frac{\partial \psi m}{\partial t} \rightarrow \text{Ind} \]

\[ \text{Ind} \]

\[ \text{Ind} \]

\[ \text{Ind} \]

Comments: Using Faraday's laws of induction, due to the rate of change of magnetic flux produced, current is induced in the circular loop, which will affect the
Induced emf when the loop is open-circuited.
Therefore induced emf equivalent circuit of $I_2$ and is always present on the loop when ever mag. field is time varying.

\[ \vec{A} = \frac{\mu}{4\pi} \int_{-\infty}^{\infty} \frac{\vec{r}}{r^2} \, dl \]

due to line current.

As $l \to \infty$, $\vec{A} = 0$

26. \[ \eta = \sqrt{\frac{mu}{\epsilon}} = \sqrt{\frac{10^6 \mu_0 \mu_2}{\epsilon_0 \delta x}} \]

27. Perpendicular \[ \frac{\mu \eta}{\epsilon} \] cond.

28. \[ \frac{\vec{E}_1}{\vec{E}_2} = \frac{\omega}{\omega} = 1 \]
\[ \frac{\vec{H}_1}{\vec{H}_2} = \frac{J_0}{J_0} \]
\[ \vec{H}_1 = \vec{H}_2 = 0 \]

29. \[ \frac{\vec{J}_c}{\vec{J}_d} = \frac{\sigma}{\omega} = 1 \]
\[ \sigma = \omega = 2\pi f - \omega \]
\[ f = \frac{\sigma}{2\pi e_0 c_0} \]
Wave propagation

1. Surface wave propagation:
   - Used at medium wave.
   - AM broadcast
   - $f = 535 \text{ kHz} - 1605 \text{ kHz}$

2. Space wave prop.
   - Ionospheric prop
     - Used at VHF,
     - Microwave range
     - Preferred for $36 \text{ GHz} - 38 \text{ GHz}$
     - TV broadcast
     - Microwave link.

   - Ionospheric
     - Used at HF range (short-wave range)
     - Preferred for FM broadcast
     - ($f = 88 \text{ MHz} - 108 \text{ MHz}$)

4. Surface wave propagation

   ![Diagram of surface wave propagation]

   **Features:**
   - Electromagnetic wave follows along the surface of the earth.

   ![Diagram of quarter-wave monopole]

   **Features:**
   - The electromagnetic wave follows along the surface of the earth.
2) The transmitting antenna is always vertically installed & therefore is always vertically polarized.

3) The electric field associated with the wave is perpendicular to the surface of earth.

4) The earth behaves as a good conductor & these form as the wave travels the electric field strength decrease exponentially.

5) The length of the antenna depends upon the frequency of operation & wavelength of operation. Quaterwave monopole are always preferred for the transmission of such signals.

6) As the frequency of operation increases the height of antenna decreases.

7) Preferred for AM broadcast on the freq. range 535 to 1605 kHz. The range of transmission can be increased only by increasing power of Transmitter.

8) Such propagation take place when the transmitting & receiving antenna very close to each other & close to surface of earth.
3. Limited range of transmission

2. Space wave prop.  
   - Teopospheric prop.  
   - LOS prop.

\[ d = 3550 \sqrt{ht + h_r} \]  
--- m

Features:
1. The space wave constitutes:
   a) direct wave
   b) ground reflected wave

3. Reflected from above ground from 30m to 200m for TV broadcast or the VHF range.

4. The e.m. wave travel from transmitter to receiver on the earth's atmosphere of a height of 10 to 15 km above.
The surface of the earth.

3. The antenna is always horizontally polarised so that electric field vector is parallel to the surface of the earth.

4. Tally pole is preferred for such projectile so that its impedance is matched with that of T.L.

5. The total range of transmission depends upon:
   1. Power of the transmitter.
   2. Height of Transmitting & Receiving Antenna.
   3. The range of transmission can be increased by increasing the height of the receiving antenna.

Factors which control the magnitude of space waves:
   1. Conductivity of earth.
   2. Permittivity & permeability of earth.
   3. Mag of the wave.
   4. Heights of Transmitting & receiving antenna.
   5. Curvature of the earth.
   6. Distance bw Transmitting & Receiving Cntn.
   7. Variation of reflecting index of earth with height.
sky wave prop
ionospheric prop

\[ \Theta = \tan^{-1} \left( \frac{d}{h} \right) \]

\( f_c = 130 \text{ MHz} \)

\( f_c = 70 \text{ MHz} \)

\( h_0 \) - depth of layer

\( S \) - skip distance
1. Used in the HFR & preheater for FM broadcast on the short range 88 MHz to 200 MHz.

2. Long range transmission is possible.

3. The range of transmission depends:
   a) take off angle.
   b) freq. of the signal.

4. The D layer has minimum electron density whereas F2 layer has max. electron density.

5. The critical freq. of a layer depends upon the electron density of the layer.
   \[ f_c = \sqrt{\frac{81}{N}} \] 
   Critical freq. Electron density (m⁻³)
   MHz

   Therefore, the critical freq. of the D layer is minimum whereas this layer has a max. value for F2 layer.

6. During night time D-layer is missing, i.e., E & F2 layer merge together to form a single layer.

7. The E-layer is the most stable layer for FM broadcast. Uses this layer for the T/f of the signal.
1) Critical Base Angle: This is the maximum base of a layer so that the wave is reflected by that layer at vertical incidence.

The e.m. wave of base less than or equal to critical base will be reflected from the layer irrespective of the angle of incidence.

As the height of layer increases, its critical base increases.

5) Skip Distance: This is the minimum distance from the Tx at the sky wave of given base, it returns to earth by the atmospheric.

The skip distance depends upon:

- Base of the wave
- $f_c$ (Critical Base)
- Height of the layer
- Change current Concentration $+N$

MUF) Maximum Usable Frequency

$$MUF = f_c \cdot \text{MUF}$$

$MUF > f_c$
\[ T_x = \text{Transmitter} \]
\[ R_x = \text{Receiver} \]

\[ \theta = \text{TOA} \]
\[ \phi = \text{angle of incidence at the layer} \]

\[ \theta + \phi = 90^\circ \]
\[ \phi = 90^\circ - \theta \]

\[ MUF = \frac{f_c \cdot \sec \phi}{\frac{\sqrt{\left(\frac{s}{2}\right)^2 + h^2}}{h}} \]

\[ MUF = f_c \cdot \sqrt{\frac{(s/2)^2 + h^2}{h}} \]

1) for a fixed location of the Tx & Rx, MUF is the frequency which makes the distance to the receiving point equal to the skip distance.

2) MUF is the frequency that gives the strongest sky wave signal at the received point.

3) The skip distance increases with the frequency of operation.
To find $E_1$, $E_d$

\[ V_1 + V_2 = 0 \]
\[ E_0(d-a) + E_d \cdot a = V_0 \]  \[ (1) \]

\[ D_1 = D_2 \]
\[ E_0 E_0 = \varepsilon_0 \varepsilon_r E_d \]
\[ E_0 = \varepsilon_2 E_d \]  \[ (2) \]

\[ \varepsilon_2 E_d (d-a) + E_d a = V_0 \]
\[ E_d = \frac{V_0}{\varepsilon_2(d-a) + a} \]
\[ E_0 = \varepsilon_2 \cdot E_d = \frac{\varepsilon_2 V_0}{\varepsilon_2(d-a) + a} = \frac{\varepsilon_2 V_0}{\varepsilon_2(d-a) + a} \]

\[ C = \frac{\varepsilon_0}{d-a} \]
\[ C = \frac{\varepsilon_0}{\varepsilon_0 + \varepsilon_r d} \]
\[ C = \frac{\varepsilon_0}{d-a} \]
\[ C_d = \frac{\varepsilon_0 \varepsilon_r A}{a} \]
\[ e \text{ is varying linearly w.r.t. } x \]

at \( x = 0 \); \( e = e_1 \text{ min.} \)

\( x = d; \ e = e_2 \text{ max.} \)

\[ V = V_0 \quad V = 0 \]

\[ x = d \quad \text{To find} \quad \text{Capacitance} \]

\[ e = e_1 + \frac{e_2 - e_1}{d} x \]

\[ \phi_s = \frac{D_0}{e} \]

\[ V = \int_{x=0}^{x=d} E \cdot dx \]

\[ E = -\nabla V \]

\[ \frac{dx}{d} \]

\[ = -\int_{x=0}^{x=d} E \cdot dx \]

\[ = -\int_{x=0}^{x=d} \frac{\phi_s}{e_1 + \frac{e_2 - e_1}{d} x} dx \]

\[ = \left[ \frac{-e_s}{e_2 - e_1} \ln \left( e_1 + \frac{e_2 - e_1}{d} x \right) \right]_{x=0}^{x=d} \]

\[ = \frac{-e_s d}{e_2 - e_1} \ln \left( e_1 + \frac{e_2 - e_1}{d} d \right) \]

\[ = \frac{-e_s d}{e_2 - e_1} \ln \left( \frac{e_2}{e_1} \right) \]

\[ = \frac{e_s d}{e_2 - e_1} \ln \left( \frac{e_2}{e_1} \right) \]
\[ c = \frac{Q}{V} = \frac{\varepsilon_s}{\varepsilon} \quad \text{Cap. per unit surface} \quad \text{f/m}^2 \]

\[ c = \frac{\varepsilon_2 - \varepsilon_1}{d \ln \left( \frac{\varepsilon_2}{\varepsilon_1} \right)} \quad \text{f/m}^2 \]
30. \[ \tan S = \frac{\sigma}{\nu e} \] 

\[ \frac{1}{2\pi f} \]

\[ \tan S = \frac{\sigma}{\nu e} \]

\[ \frac{1}{2\pi f} \]

\[ 1.35 \]

31. \[ e_0 = Dn = e \in E_0 \]

\[ R < 2V/m \]

\[ 80 \leq e_0 \]

32.

\[ I_a \]

\[ \frac{R_{x-1}}{P_2} \]

\[ \frac{R_{x-2}}{P_1} = \frac{1}{2} \]

\[ \sigma_1 = 5 \text{ km} \]

\[ \sigma_2 \]

\[ d = x_2 - x_1 \]

\[ = (x_2 - 5) \text{ km} \]

\[ |P| = \frac{\text{power}}{4\pi \sigma^2} \]

\[ |P| \alpha \frac{1}{\sigma^2} = p \]

\[ \frac{P_2}{P_1} = \left( \frac{\sigma_1}{\sigma_2} \right)^2 = \frac{1}{2} \]

\[ \Rightarrow \sigma_2 = \sqrt{2} \sigma_1 \Rightarrow 5\sqrt{2} \text{ km} \]

\[ \Rightarrow x_2 - x_1 = d = (5\sqrt{2} - 5) = 5(\sqrt{2} - 1) = 2.070 \text{ km} \]

\[ P_{dB} = 10 \log_{10} P \]

\[ (P_2) = -3 \text{ dB} = 10 \log_{10} (P_2/P_1) \]

\[ \Rightarrow \frac{P_2}{P_1} = \frac{1}{2} \]
\[ e = e_0 \]
\[ A \]
\[ v = 0.5 \nu \]
\[ f = 3.6 \text{ GHz} \]
\[ I_d = \frac{2D \cdot A}{\partial t} \cdot A = j\omega E A = j\omega e_0 \frac{V}{d} \]
\[ A/m^2 \]
\[ |I_d| = 2\pi f e_0 \frac{V}{d} A \]

Tensor \( \vec{\rho} = \frac{1}{2} \text{Re} \left[ \vec{E} \times \vec{H}^\star \right] \)

\[ \vec{\rho} = \frac{1}{2} \text{Re} \left[ (a_x + j a_y) e^{j k x} e^{j w t} (K_{\text{mu}}) (a_y + j a_x) e^{j k y} (H_{\text{mu}}) \right] \]

\[ K' \text{Re} \left[ a_x - a_y \right] \]

\[ = 0 \quad \text{null vector} \]

1. \( \vec{P} = \vec{E} \times \vec{H} \)
2. \( \vec{P} = \frac{1}{2} \vec{E} \times \vec{H} \)
3. \( \vec{P} = \frac{1}{2} \text{Re} \left[ \vec{E} \times \vec{H}^\star \right] \)

--- Poynting's vector
--- average Poynting vector
--- average Poynting vector when \( \vec{E} \times \vec{H} \) are phasors.
\[ E = 2 \hat{a}_x \]
\[ H = 4 \hat{a}_y \]
\[ \hat{p} = E \times H = 8 \hat{a}_z \]

\[ E = 4 \cos(\omega t - \beta z) \hat{a}_x \]
\[ H = 2 \cos(\omega t - \beta z) \hat{a}_y \]
\[ \hat{p} = (E \times H) = 8 \cos^2 \left( -\frac{\omega}{2}\right) \hat{a}_z \]

\[ p_{av} = \frac{1}{2} 8 \hat{a}_z. \]

\[ E = (\hat{a}_x + j \hat{a}_y) e^{jkz - j\omega t} \]

\[ H = \quad \text{phase}. \]

\( \delta : 39 \)

\[ \text{Med (1)} : \text{free space} \quad \varepsilon_0 > \varepsilon_0 \]
\[ \text{Med (2)} : \text{perfect dielectric} \quad (\sigma = 0, \mu_e = 1) \]
\[ \varepsilon_2, \varepsilon_2 : \varepsilon_1, \varepsilon_0 \]

\[ \epsilon > \varepsilon_0 \]

\[ S = \frac{V_{max}}{V_{min}} = \frac{E_{max}}{E_{min}} = 5 \]

\[ p = \frac{S-1}{S+1} = \frac{5-1}{5+1} = \frac{2}{3} \]

\[ p = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta - \eta_0}{\eta + \eta_0} \]

\[ \rho = \frac{\varepsilon_0 - \varepsilon}{\varepsilon_0 + \varepsilon} < \frac{\sqrt{\varepsilon - \varepsilon_0}}{\sqrt{\varepsilon + \varepsilon_0}} < \frac{\varepsilon_0 - \varepsilon}{\varepsilon_0 + \varepsilon} \]

\( \rho \geq 120 \pi \)
\[ \eta = 24 \pi \ \text{dB} \]

\[ p_i = p_e + p_t \]

\[ l = \frac{p_e}{p_i} + \frac{p_t}{p_i} \]

\[ \left( \frac{p_t}{p_i} \right) = \left[ 1 - \left( \frac{p_e}{p_i} \right) \right] \]

Reflection coefficient of power.

\[ \frac{p_t}{p_i} = 1 - \left( \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}} \right)^2 \]

\[ \epsilon_1 = \epsilon_0 \epsilon_1 = \epsilon_0 \]

\[ \epsilon_2 = \epsilon_0 \epsilon_2 = 4 \epsilon_0 \]

\[ \frac{p_t}{p_i} = \frac{8}{9} \]

\[ E_z = E_0 e^{-\gamma z} \]

Along +z direction

\[ E_z = E_0 e^{-(y + j\beta z)} \]

Toss less Med.

\[ E_z = E_0 e^{-j\beta z} \]

\[ E_z = E_0 e^{-j\beta z} e^{j\omega t} \]

for sinusoidally varying field.

\[ E_z = E_0 e^{j(\omega t - \beta z)} \]

+ z direction prop.

\[ \rightarrow \text{Wave propagates along some arbitrary direction} \]

\[ E_z = E_0 e^{j(\omega t - \beta z)} \]

To be ignored.
\[ \vec{B} \cdot \vec{\sigma} = (B_x \hat{\alpha} + B_y \hat{\alpha} + B_z \hat{\alpha}) \cdot (x \hat{\alpha} + y \hat{\alpha} + z \hat{\alpha}) = B_x x + B_y y + B_z z \]

\[ = B \cos 0^\circ + B \cos 0^\circ + B \cos 0^\circ \]

\[ = \frac{2\pi \cos 0^\circ}{\delta} = 0^\circ \]

\[ = \frac{\sqrt{3}}{2} \pi \cdot \frac{1}{a} = \frac{\pi}{a} \]

\[ E_z = E_0 \cdot e^{j(\omega t - \frac{\sqrt{3} \pi r}{a} - \frac{\pi}{a} \cdot \tau)} \]

\[ \text{Incident :} \quad RCP \]

\[ \text{Reflected :} \quad LP \]

\[ \tan \alpha_B = \frac{c_2}{c_1} \rightarrow c_2 \cdot c_0 \]

\[ \tan \alpha_B = \sqrt{\frac{c_2}{c_1}} \rightarrow c_2 \cdot c_1 = c_0 \]

\[ c_0 \cdot c_1 = 3 \]

\[ \epsilon_{11} = 1 \quad \epsilon_{1} = \epsilon \frac{a_x}{a} \quad \epsilon_{2} = \epsilon_2 \frac{a_x}{a} \]

\[ \epsilon = D_{11} - D_{12} \]

\[ = \epsilon_0 \epsilon_{11} E_{11} - \epsilon_0 \epsilon_{22} E_{12} \]

\[ = -3 \epsilon_0 \]

\[ = \epsilon_0 [1 - 2 \times 2] \]
End both call

Perfect cond.

\[ V \]

\[ V_{losses} = 0 \]

\[ \text{Heat dissipation} = 0 \]

\[ \vec{H} = 0.1 \cos \left( 4 \times 10^7 t - \beta z \right) \hat{a}_2 \]

\[ \text{Proof} \rightarrow +3 \]

\[ \vec{P} = \vec{E} \times \vec{H} \]

\[ \hat{a}_2 \times \left( -\hat{a}_y \right) \times \left( \hat{a}_x \right) \]

\[ \vec{E} = -E_y \hat{a}_y \]

Notice

\[ \eta \text{Ohm} = 3.77 \times 0.1 \]

\[ \vec{E} = 50 \cos \left( \frac{\pi}{12} \right) \hat{a}_x \]

\[ \vec{H} = \frac{5}{12 \pi} \cos \left( \frac{\pi}{9} \right) \hat{a}_y \]

\[ \text{Directed on prof.} \rightarrow +3 \]

Power \[ = p_3 \times v \times e \times q_1 \]

\[ \frac{1}{2} \epsilon m H_m \left( \eta \frac{\tau^2}{2} \right) \]

\[ 50 \times \frac{5}{12 \pi} \]
\[ H = (\ldots) \cdot dA \]

\[ E \neq dA \]

-- wave is not polarized in \( z \) direction

\[ (F/dl) = I \times B \]

\[ E \text{ind} \begin{cases} \text{Cond. as moving} (\vec{v}^*) : \vec{B} \neq f(t) \\ \vec{v}^* = 0 : \vec{B} = f(t) \\ \vec{v}^* ; \vec{B} = f(t) \end{cases} \]

**Case 1:** Moving cond. \( \vec{v}^* \)

\[ E \text{ind} = \int_c (\vec{v}^* \times \vec{B}) \cdot dl \]

--- Motional emf.

**Case 2:** \( \vec{v}^* = 0 \)

\[ \vec{B} = f(t) \]

\[ E \text{ind} = \int \int_S \frac{\partial \vec{B}}{\partial t} \cdot dA \]  

--- due to time varying field

**Case 3:** Moving cond. \( \vec{B} \)

\[ E \text{ind} = E \text{ind (case 1)} + E \text{ind (case 2)} ]

--- emf due to transformer action
\( \sigma \) \( \epsilon \) \( \varepsilon_s \)

\[
p = \rho \omega = \frac{1}{2} \epsilon E_m^2 = \frac{1}{2} \left( \frac{\varepsilon_s}{\epsilon} \right) = \frac{1}{2} \epsilon \frac{\varepsilon_s^2}{\epsilon_s} - \frac{\varepsilon_s^2}{\epsilon} = \frac{1}{2} \varepsilon \sigma_0^2
\]

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\[ PE = KE \]

Potential energy = Kinetic energy

\[ eV = \frac{1}{2} m v^2 \]

\[ v = \sqrt{\frac{2 eV}{m}} \quad \text{Known} \]

\[ f = \frac{mv^2}{r} = e \frac{v^2}{B} \]

\[ E = m v^2 \]

Result:

250 Gauss \rightarrow \text{to convert}

\[ 250 \times 10^{-4} \]

\[ E = \frac{vB}{\ell A} \]

\[ H = \frac{1}{\mu_0} \]

\[ \frac{HA}{AB} = 9 = \frac{vB}{\ell A} \]
\[
\frac{d}{dx} = 2
\]

\[
E = \vec{\dot{y}} \cdot A \cos \omega t - \frac{\varepsilon_0}{\mu_0} \cdot \vec{\dot{y}}
\]

\[
\frac{\varepsilon_0}{\mu_0} \cdot \vec{\dot{y}}
\]

\[
\frac{E}{\varepsilon_0} = \frac{E_z}{\varepsilon_0} \cdot \dot{y}
\]

\[
\vec{P} = \vec{E} \times \vec{H}
\]

\[
\vec{A} = \vec{\dot{y}} \times (-\alpha x)
\]

\[
\frac{H^2}{\varepsilon_0} = \frac{\varepsilon_0}{\mu_0} \cdot \alpha x
\]

\[
\frac{E}{\varepsilon_0} = \frac{E_z}{\varepsilon_0} \cdot \dot{y}
\]

\[
\frac{\varepsilon_0}{\mu_0} \cdot \vec{\dot{y}}
\]

\[
\alpha x \left[ \begin{align*}
\vec{\dot{y}} & = \frac{\varepsilon_0}{\mu_0} \cdot A \cos \omega t - \frac{\varepsilon_0}{\mu_0} \cdot A \sin \omega t \\
\vec{\dot{y}} & = \cos \omega t - \sin \omega t
\end{align*} \right]
\]

\[
\vec{D} = \frac{\varepsilon_0}{\mu_0} \cdot (\alpha x - \sqrt{3} \alpha y)
\]

\[
|\vec{D}| = 2 \sqrt{1 + 3} = 4 \cdot C/m^2
\]

\[
|\vec{D}| = \varepsilon_0 = 4 \cdot C/m^2
\]

\[
+ \quad + \quad + \quad + \quad + \quad + \quad + \quad +
\]
\[ \Phi = \frac{q^2}{4\pi \varepsilon_0 d^2} \]

\[ d \downarrow \quad f_y = mg \]

\[ mg = \frac{q^2}{4\pi \varepsilon_0 d^2} \quad \text{bind } d = 8.57 \text{ cm} \]

\[ \Phi = -\frac{6 \varepsilon_0}{\varepsilon} \]

\[ \text{bind } E = -\nabla \Phi \]

\[ \text{bind } D = \varepsilon \varepsilon_0 \]

\[ \text{bind } \varepsilon = \nabla \cdot D \]

\[ \text{bind } \Phi = \iiint q = \Phi \partial \Omega \]

\[ q = -\varepsilon_0 \nabla^2 \Phi ; \quad \Phi = f(t) \]

\[ = -\varepsilon_0 \frac{1}{\varepsilon^2 \sin \theta} \frac{\partial}{\partial \varepsilon} \left( \varepsilon^2 \sin \theta \frac{\partial \Phi}{\partial \varepsilon} \right) \]

\[ = -\varepsilon_0 \frac{1}{\varepsilon^2} \frac{\partial}{\partial \varepsilon} \left[ -\frac{\varepsilon^2}{\varepsilon_0} \left( \frac{6 \varepsilon - 30}{\varepsilon} \right) \right] \]

\[ = +\varepsilon_0 \frac{1}{\varepsilon^2} \left[ \frac{\varepsilon^2}{\varepsilon_0} \left( \frac{6 \varepsilon - 30}{\varepsilon} \right) \right] \]

\[ = \frac{1}{2} \times 30 \times 6 \times 5 \]

\[ = 180 \varepsilon^3 \]
\[ \alpha = \iiint_V e \, dv \]
\[ = 180 \int_0^{1/6} \int_0^1 \int_0^{2\pi} r^5 \, dr \, d\phi \, d\theta \]
\[ = 180 \int_0^{1/6} \int_0^1 \theta \, d\theta \, d\phi \]
\[ = 180 \times \frac{1}{6} \times 4\pi \]
\[ = 120\pi \text{ Ans} \]

\[ \text{Q. 7} \quad \vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z} \]

\[ \nabla \times \vec{A} = 0 \quad \text{irrotational vector} \]

\[ \nabla \cdot \vec{A} \neq 0 \quad \text{not solenoidal} \]

\[ \text{Divergence less} \]

\[ \text{Q. 8} \quad \text{We} = q_1 \phi_2 \]

\[ \frac{q_1}{4\pi \varepsilon_0} \]

\[ \text{We} \propto \frac{1}{\varepsilon} \]

\[ \text{We}_2 = \frac{1}{2} \text{We}_1 \]

\[ \text{Q. 11} \quad \text{Line charge} = \left( y=3; z=5 \right) \]

\[ E(0, 6; 1) = \quad \quad E(5, 6; 1) = ? \quad \text{remains same} \]
\[ V = -\int E \cdot d\mathbf{l} \]

\[ = -\left( \int_A x \, dx + \int_B y \, dy + \int_C z \, dz \right) \]

\[ = +5 \text{ Volts} \]

\[ V = 3\varepsilon_0 j - j3 \]

\[ V(1,1,0,1) = 0 \]

\[ \nabla^2 V = -\nabla \cdot \nabla V = - \left[ \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \right] \]

\[ \neq 0 \]

Polarization in dielectric materials:

- Free charge \( \rho \) is \( \text{C/m}^2 \)
- Present on conductor
- Bound charge \( \sigma \)
- Surface charge density due to polarization

\[ D = \varepsilon_0 E \quad \text{for free space} \]
\[ D = \varepsilon_0 (\varepsilon E) \quad \text{for dielectric} \]
\[ D = \varepsilon_0 E + P \quad \text{for dielectric} \]
\[ D = \varepsilon_0 E + p \]
\[ p = D - \varepsilon_0 E = D - \varepsilon_0 \frac{D}{\varepsilon_0} = D - \frac{D}{\varepsilon_0 \varepsilon_r} \]

\[ p = D \left(1 - \frac{1}{\varepsilon_r}\right) = 2 \left(1 - \frac{1}{5}\right) = 1.6 \times 10^{-6} \text{ C/m}^2 \]

\( \text{imp} \) ① The polarization on the dielectric material is present whenever it is subjected to some externally applied electric field.

② The charges are induced with on the dielectric due to dipole orientation so that net charge induced on the dielectric slab is zero.

③ Polarization represents total dipole moment orientation per unit volume of the dielectric.

④ Due to induced charges, overall field distribution on the dielectric is modified.

⑤ It is a temporary phenomenon so long as externally applied electric field is present the dielectric will remain in the polarized stage.

As soon as this electric field with drawn the dielectric returns to its unpolarized stage and the induced charges are no longer present.
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\[ \vec{F}_{13} + \vec{F}_{23} = 3 \vec{F} \quad -1 \]

\[ \vec{F}_{12} + \vec{F}_{32} = 2 \vec{F} \quad -2 \]

\[ \vec{F}_{31} + \vec{F}_{31} = \vec{0} \]

\[ -\vec{F}_{31} - \vec{F}_{32} = 3 \vec{F} \]

\[ -\vec{F}_{21} + \vec{F}_{32} = 2 \vec{F} \]

\[ \vec{F}_{31} + \vec{F}_{21} = -5 \vec{F}, \text{ Ans.} \]

Q. 122

\[ \vec{E} = \vec{Q} \times \vec{B} \]

\[ \vec{E} = -\vec{v} \times \vec{B} \]
1) cart. coord. system: 
\[(x, y, z)\]

\[d\mathbf{S} = \begin{cases} 
  + dx \, dy \, dz & \text{if } x \text{ and } y \text{ are positive} \\
  - dx \, dy \, dz & \text{if } x \text{ and } y \text{ are negative} \\
  \pm dy \, dz \cdot \hat{a}_x & \text{if } x \text{ is positive and } y \text{ is negative} \\
  \pm dy \, dz \cdot \hat{a}_z & \text{if } x \text{ is negative and } y \text{ is positive} \\
\end{cases} \]

\[d\mathbf{v} = \hat{a}_x \cdot dx \cdot dy \cdot dz \]
\[\begin{cases} 
  -\infty < x < +\infty \\
  -\infty < y < +\infty \\
  -\infty < z < +\infty 
\end{cases} \]

2) cyl. coord. system: 
\[(r, \phi, z)\]

\[d\mathbf{S} = \begin{cases} 
  + r \, dr \, d\phi \, dz & \text{if } r \text{ and } \phi \text{ are positive} \\
  - r \, dr \, d\phi \, dz & \text{if } r \text{ and } \phi \text{ are negative} \\
  \pm r \, dz \cdot d\phi \cdot \hat{a}_z & \text{if } r \text{ is positive and } \phi \text{ is negative} \\
  \pm r \, dz \cdot d\phi \cdot \hat{a}_z & \text{if } r \text{ is negative and } \phi \text{ is positive} \\
\end{cases} \]
$$dv = \pi dz \, d\phi \, d\xi$$

\[
\begin{cases}
0 \leq \xi < \infty \\
0 \leq \phi < 2\pi \\
-\infty < \zeta < +\infty
\end{cases}
\]

Sph. Coord. System: \((\zeta, \Theta, \Phi)\)

\[
d\overrightarrow{r} = \begin{cases}
\pm \zeta \, d\zeta \, \hat{\zeta} \\
\pm \zeta \sin \theta \, d\theta \, \hat{\theta} \\
\pm \zeta \sin \theta \, \sin \phi \, d\phi \, \hat{\phi}
\end{cases}
\]

\[
d\overrightarrow{s} = \begin{cases}
\pm \zeta^2 \, d\zeta \, d\phi \, \hat{\phi} \\
\pm \zeta^2 \, \sin \theta \, d\theta \, d\phi \, \hat{\phi} \\
\pm \zeta^2 \, \sin \theta \, \sin \phi \, d\phi \, d\theta \, \hat{\phi}
\end{cases}
\]

\[
dv^2 = \zeta^2 \sin \theta \, d\zeta \, d\theta \, d\phi
\]

\[
\begin{cases}
0 \leq \zeta < \infty \\
0 \leq \theta < \pi \\
0 \leq \phi < 2\pi
\end{cases}
\]
General relations

\[(u, v, w)\]

1. \( \nabla v = \sum \frac{1}{h_i} \frac{\partial v}{\partial u} \hat{a}_\mu \)

2. \( \nabla \cdot \vec{A} = \frac{1}{h_1 h_2 h_3} \sum \frac{\partial}{\partial \mu} \left( h_2 h_3 A_\mu \right) \)

3. \( \nabla^2 v = \frac{1}{h_1 h_2 h_3} \sum \frac{\partial}{\partial \mu} \left( \frac{h_2 h_3}{h_1} \frac{\partial v}{\partial \mu} \right) \)

4. \( \nabla \times \vec{A} = \frac{1}{h_1 h_2 h_3} \left[ \begin{array}{ccc} h_1 \frac{\partial u}{\partial u} & h_2 \frac{\partial v}{\partial u} & h_3 \frac{\partial w}{\partial u} \\ \frac{\partial v}{\partial \mu} & \frac{\partial w}{\partial \mu} & \frac{\partial u}{\partial \mu} \\ -h_1 A_\mu & -h_2 A_\nu & -h_3 A_\omega \end{array} \right] \)

\[ \vec{A} = A_\mu \hat{a}_\mu + A_\nu \hat{a}_\nu + A_\omega \hat{a}_\omega \]

<table>
<thead>
<tr>
<th>rest.</th>
<th>x</th>
<th>y</th>
<th>z</th>
<th>( h_1 )</th>
<th>( h_2 )</th>
<th>( h_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>cyl.</td>
<td>( \xi )</td>
<td>( \phi )</td>
<td>( z )</td>
<td>1</td>
<td>( \xi )</td>
<td>1</td>
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<tr>
<td>sph.</td>
<td>( \xi )</td>
<td>( \theta )</td>
<td>( \phi )</td>
<td>1</td>
<td>( \theta )</td>
<td>( \sin \theta )</td>
</tr>
</tbody>
</table>

Smith chart

T. L. Calculate
1. Lossless Line
2. Normalized impedance

\( \frac{Z}{Z_0} = \bar{Z} = \bar{R} \pm j \bar{X} \)
1. The Smith chart represents const. resistance & const. reactance circle, which are orthogonal at each point.

2. The line is assumed as lossless & the admittance is always plotted on its normalized form.

3. The total circumference of the chart is equal to \( \pi/2 \) or length of each half circle of the chart; represent a distance of \( \pi/4 \).

4. The leftmost point represents voltage minima whereas the rightmost point of the chart represent voltage maxima.

5. The centre point of the chart corresponds to a matched line when \( Z_L = Z_0 \).

6. The total distance below the centre of the chart to the rightmost point represents total range of VSWR from 1 to \( \infty \).
7) Upper half the circle corresponds to active reactance whereas lower half of the circle represents negative reactance.

8) To find the normalized admittance from normalized Z0, a distance of \( \pi / 2 \) is move along the constant VSWR circle.

9) Going clockwise in the chart, the impedance one moves towards the generator & sourced impedances are added to the initial impedance.

10) Going anticlockwise or towards the load, capacitive reactance is added to the initial impedance along the line.

\[
\begin{align*}
Z_{in} &= Z_{sc} = j \omega L \tan \theta \\
\text{if } \theta &= \pi / 4 \\
Z_{sc} &= j \omega L \tan \frac{2\pi - \theta}{4} \\
&= j\infty
\end{align*}
\]

Parallel LC Resonance

\[
Z_{oc} = -j\omega C \tan \theta | \theta = \pi / 4 \\
= 0
\]
Calculation 1

(1) For 1/4 section of the line a short circuit line represent a parallel LC resonant circuit, whereas as an open circuit line represent a series LC resonant circuit.